We are asked to find the interval of convergence for

$$\sum_{n=0}^{\infty} \frac{2^n}{n3^n} x^{2n}$$

and to determine where convergence is absolute or conditionally.

First, the easy part. Since x is always raised to an even power, the series converges absolutely anywhere it converges. Several students wrote that this also means it converged conditionally, but this is a mistake. By definition, a series can only converge conditionally if it does not converge absolutely.

Now we determine the radius of convergence. The easiest method is to apply the root test. Clearly,

$$\sqrt[n]{\frac{2^n}{n3^n} x^{2n}} = \frac{2x^2}{3\sqrt[n]{n}}$$

Using the fact that $\lim_{n\to\infty} \sqrt[n]{n} = 1$, we have that the limit of the n-th root of the terms is $2x^2/3$. In order for the series to converge, we must have

$$\frac{2x^2}{3} \leq 1$$

which means

$$|x| \leq \sqrt{3}/\sqrt{2}$$

We would arrive at the same conclusion if we applied the ratio test.

Most of the mistakes in this part of the problem were algebraic. Some did not know how to solve for x and arrived at numbers such as $\sqrt{-\frac{3}{2}}$, which is purely imaginary. Many forgot to take square roots at all. These mistakes were docked according to their severity and according to whether they showed a fundamental misunderstanding of the material.

Now we need to check the endpoints. Since the series is always positive, it is immaterial whether we check $\sqrt{3}/\sqrt{2}$ or its negative. We will check $-\sqrt{3}/\sqrt{2}$ because many students thought this resulted in an alternating series.

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{2^n}{n3^n} \left(\frac{-\sqrt{3}}{\sqrt{2}}\right)^{2n} &= \sum_{n=0}^{\infty} \frac{2^n}{n3^n} \frac{3^n}{2^n} \\ &= \sum_{n=0}^{\infty} \frac{1}{n}\end{aligned}$$

This is the harmonic series, which diverges.

There were many mistakes in checking the endpoints. The most common were not checking them at all, believing that the series could be alternating, or performing other algebraic steps incorrectly.

The grading was split up as follows:

1. 10 points for finding the radius.

2. 5 points for checking the endpoints of the interval
3. 3 points for getting absolute/conditional convergence
4. 2 points for writing down the interval properly

Additional points were taken off for writing down false statements or otherwise showing a misunderstanding of the material. (For example, writing an equals sign between two expressions that are obviously not equal). The one exception is that the series above is actually defined at 0, which is a typo on the exam. No points were taken off for students trying to deal with this typo, regardless of whether what they wrote was mathematically correct.