

Midterm 2

3) Determine whether the function  $f(x) = x + \sin(x)$  has an inverse. If so, give the domain and range.

(5 points)  $f(x) = x + \sin(x)$ , so  $f'(x) = 1 + \cos(x)$ .

(15 points)  $f'(x) \geq 0$  and  $f'(x) = 0$  for  $x = \pi + 2k\pi$  (for integer  $k$ ) which is a discrete set and hence  $f$  is increasing. Thus,  $f$  has an inverse.

(10 points) The domain and range of  $f^{-1}$  are the range and domain (resp) of  $f$ . The domain of  $f$  is all reals, hence so is the range of  $f^{-1}$ . Also, note that since  $\lim_{x \rightarrow -\infty} f(x) = -\infty$ ,  $\lim_{x \rightarrow \infty} f(x) = \infty$ , and  $f$  is continuous, the range of  $f$  is all reals, hence so is the domain of  $f^{-1}$ .