MATH 141, FALL 2013, MIDTERM 4
Problem 4

a) Write

$$\frac{-2\sqrt{3}+i2}{-1+i\sqrt{3}} \tag{1}$$

in the polar form $re^{i\theta}$.

$$\frac{-2\sqrt{3}+i2}{-1+i\sqrt{3}} = \frac{-2\sqrt{3}+i2}{-1+i\sqrt{3}}\frac{(-1-i\sqrt{3})}{(-1-i\sqrt{3})} = \sqrt{3}+i \tag{2}$$

(5 pt.)

$$r = \sqrt{(\sqrt{3})^2+(1)^2} = 2 \tag{3}$$

(3 pt.)

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \tag{4}$$

(3 pt.)

$$\theta = \frac{\pi}{6} \tag{5}$$

(2 pt.)

$$z = 2e^{i\frac{\pi}{6}} \tag{6}$$

(2 pt.)

b) Find all fifth roots of unity, i.e., all complex numbers z such that $z^5 = 1$.

After expressing z in its polar form, $z = re^{i\theta}$, we have

$$z^5 = (re^{i\theta})^5 = r^5 e^{i5\theta} = 1 = 1\cdot e^{i(2\pi n)} \tag{7}$$

with $n \in \mathbb{N}$. That is, 5θ must be a multiple of 2π.
Therefore

$$r = 1 \tag{8}$$

(3 pt.)

and

$$\theta = \frac{2\pi}{5}k \tag{9}$$

(3 pt.)

with $k = 0, 1, 2, 3, 4$.

The five fifth roots of 1 are:

$$z_0 = 1 \cdot e^{i\frac{2\pi}{5}\cdot 0} = \cos(\frac{2\pi}{5} \cdot 0) + i\sin(\frac{2\pi}{5} \cdot 0) = 1 \tag{10}$$

$$z_1 = 1 \cdot e^{i\frac{2\pi}{5}\cdot 1} = \cos(\frac{2\pi}{5} \cdot 1) + i\sin(\frac{2\pi}{5} \cdot 1) \tag{11}$$

$$z_2 = 1 \cdot e^{i\frac{2\pi}{5}\cdot 2} = \cos(\frac{2\pi}{5} \cdot 2) + i\sin(\frac{2\pi}{5} \cdot 2) \tag{12}$$

$$z_3 = 1 \cdot e^{i\frac{2\pi}{5}\cdot 3} = \cos(\frac{2\pi}{5} \cdot 3) + i\sin(\frac{2\pi}{5} \cdot 3) \tag{13}$$

$$z_4 = 1 \cdot e^{i\frac{2\pi}{5}\cdot 4} = \cos(\frac{2\pi}{5} \cdot 4) + i\sin(\frac{2\pi}{5} \cdot 4) \tag{14}$$

(4 pt.)