

Solution of Problem 5

$$f'(x) = \frac{1}{(2+x)^2} \quad (5 \text{ pts})$$

$$f''(x) = \frac{-2}{(2+x)^3}$$

$$f'''(x) = \frac{6}{(2+x)^4}$$

\vdots

$$f^{(k)}(x) = \frac{(-1)^{k+1}k!}{(2+x)^{k+1}} \quad (5 \text{ pts})$$

$$\frac{f^{(k)}(-1)}{k!} = (-1)^{k+1} \quad (3 \text{ pts})$$

$$\text{Final answer} : \sum_{k=1}^{\infty} (-1)^{k+1} (x+1)^k \quad (5 \text{ pts})$$