Pulse propagation in sea water

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The propagation in sea water of a low-frequency electromagnetic pulse generated by an electric dipole is investigated analytically. The dipole is excited by a rectangular current pulse with a finite, nonzero rise and decay time. In order to obtain an explicit formula for the field in the equatorial plane of the dipole source that is uniformly valid in distance and time, Fourier-transform methods are applied. Certain limiting forms of the current pulse are studied separately. Simple analytic expressions of the field are obtained, compared to previous results, and examined thoroughly. The effect of the finite rise and decay time is discussed. It is noted that the present analysis may be used for studying pulse propagation in any highly conducting medium besides sea water. © *1995 American Institute of Physics.*

I. INTRODUCTION

When a pulse generated by the current in an electric dipole travels in a dissipative medium, its shape along with its characteristics (amplitude, duration, rise and decay time) are modified. This is mainly due to the fact that the wave number is no longer linear in frequency and that the dipole source creates a field of interest which involves the complete near, intermediate, and far fields. As a consequence, the form of the propagating pulse shifts successively from that of the excitation current and near field to its spatial and its time derivatives.[1–3]

In this paper, the propagation of a pulse with a nonzero rise and decay time is investigated; its bandwidth in the frequency domain is assumed to be narrow and centered at the origin, i.e., the signal is not modulated. Despite this restriction, the results may be used to obtain physical insight into the case of a sinusoidally modulated electric-current pulse with a similar envelope. In this case, the transients of the propagating pulse depend on the form of the envelope of the excitation current and ultimately dominate the advancing wavepacket.[2,3] Furthermore, realistic pulses do not extend from $-\infty$ to $+\infty$ in time[1] nor do they exhibit step discontinuities as does the ideal rectangular pulse. Therefore, it is necessary to study the effect of a nonzero rise and decay time on the transient response. Such a consideration results in the elimination of the delta function as a useful pulse.[4] It will be shown that the step discontinuities in their first derivative add a correction term to the response of the corresponding step-discontinuous pulse which is significant in times of the order of the rise or decay time. This term is evaluated exactly and agrees with previous results in the limit of a very short rise or decay time.

The low-frequency approximation is mainly based on the condition $\sigma/\omega\epsilon \gg 1$, valid for all frequencies of interest in sea water. A similar approach may be employed for other conducting media besides sea water, provided the aforementioned condition is satisfied.

II. DEFINITION OF THE CURRENT PULSE AND ITS FOURIER TRANSFORM

A normalized rectangular pulse with a nonzero rise and decay time can be expressed in terms of the Heaviside step function $u(t)$ as follows:

$$f(t)=\frac{1}{2t_1}\{(1-e^{-\omega_p t})u(t)-[1-e^{-\omega_p(t-2t_1)}]u(t-2t_1)\}, \tag{1}$$

where $2t_1$ is the width of the original rectangle and $\tau_p=1/\omega_p$ is the rise time, which is taken to be equal to the decay time.

The electric dipole is excited by a current

$$I_z(t)=I_0 f(t). \tag{2}$$

The Fourier transform of this pulse is

$$\tilde{I}_z(\omega)=\int_{-\infty}^{\infty} I_z(t)e^{i\omega t}dt=\frac{I_0}{2\omega t_1}\frac{\omega_p}{\omega+i\omega_p}(e^{2i\omega t_1}-1). \tag{3}$$

It is worth noting that in the limit $\omega_p\to+\infty$, $\tilde{I}_z(\omega)$ reduces to

$$\tilde{I}_z(\omega)=I_0 e^{i\omega t_1}\frac{\sin(\omega t_1)}{\omega t_1}, \tag{4}$$

which is the Fourier transform of the normalized ideal rectangular envelope

$$I_z(t)=\frac{I_0}{2t_1}[u(t)-u(t-2t_1)]. \tag{5}$$

In the limit $t_1\to 0^+$, Eq. (3) reduces to

$$\tilde{I}_z(\omega)=I_0\frac{\omega_p}{\omega_p-i\omega}, \tag{6}$$

which is the Fourier transform of the normalized exponential pulse

$$I_z(t)=I_0\omega_p e^{-\omega_p t}u(t). \tag{7}$$

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In the above, limit means "limit in the mean" (l.i.m.) in the metric space $L^2(-\infty,+\infty)$ of the square integrable functions. Both of these pulses reduce to the Dirac delta function in the limit $t_1\to 0^+$ or $\omega_p\to+\infty$, respectively, according to the Dirac measure definition in distribution theory.

III. THE ELECTRIC FIELD AND ITS TRANSFORM

The $\hat{z}$-directed, frequency-dependent electric field generated by an electrically short dipole with its axis along the z axis and an electric moment $2h_eI_0$ is given by Eq. (15) of Ref. 3, viz.,

$$\tilde{E}_z(\rho,\omega)=\frac{\mu_0 a h_e \tilde{I}_z(\omega)}{2\pi}\left(\frac{i\omega}{a\rho}+\frac{(i-1)\sqrt{\omega}}{2a^2\rho^2}-\frac{1}{2a^3\rho^3}\right)\times e^{-a\rho\sqrt{\omega}}e^{ia\rho\sqrt{\omega}} \qquad (8)$$

on the plane $z=0$ perpendicular to the dipole, where $\rho=(x^2+y^2)^{1/2}$ is the distance from the center of the dipole, and

$$a=(\mu_0\sigma/2)^{1/2}. \qquad (9)$$

In Eq. (8), the complex wave number

$$k(\omega)=\omega[\mu_0(\epsilon+i\sigma/\omega)]^{1/2} \qquad (10)$$

has been approximated by

$$k(\omega)=(i\omega\mu_0\sigma)^{1/2}=(1+i)(\omega\mu_0\sigma/2)^{1/2}=(1+i)a\sqrt{\omega}. \qquad (11)$$

Since only low frequencies are useful in sea water, the condition

$$\sigma\gg\omega\epsilon \qquad (12)$$

has been imposed as a simplifying approximation. With $\sigma\simeq 4$ S/m and $\epsilon_r\simeq 80$, condition (12) on the frequency is $f\ll\sigma/2\pi\epsilon_r\epsilon_0=9.0\times 10^8$ Hz. Since frequencies of the order of 0–100 Hz are of interest for the carrier frequency, this condition is no practical restriction.

The time-dependent electric field is

$$E_z(\rho,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{E}_z(\rho,\omega)e^{-i\omega t}d\omega=\frac{\mu_0 a' h_e I_0}{8\pi t_1^2}A(\rho',t'), \qquad (13)$$

where

$$a'=\frac{a}{\sqrt{t_1}},\quad t'=\frac{t}{t_1},\quad \omega'=\omega t_1,\quad \omega_p'=\omega_p t_1,\quad \rho'=\frac{a\rho}{\sqrt{t_1}}, \qquad (14)$$

and

$$A(\rho',t')=\frac{A_1(\rho',t')}{\rho'}+\frac{A_2(\rho',t')}{\rho'^2}+\frac{A_3(\rho',t')}{\rho'^3}, \qquad (15)$$

$$A_j(\rho',t')=I_j(\rho',t'-2)-I_j(\rho',t'),\quad j=1,2,3, \qquad (16)$$

$$I_1(\rho',t')=\frac{i}{\pi}\int_{-\infty}^{\infty}\frac{\omega_p'}{\omega'+i\omega_p'}e^{-\rho'\sqrt{\omega'}}\times e^{-i(\omega't'-\rho'\sqrt{\omega'})}d\omega', \qquad (17)$$

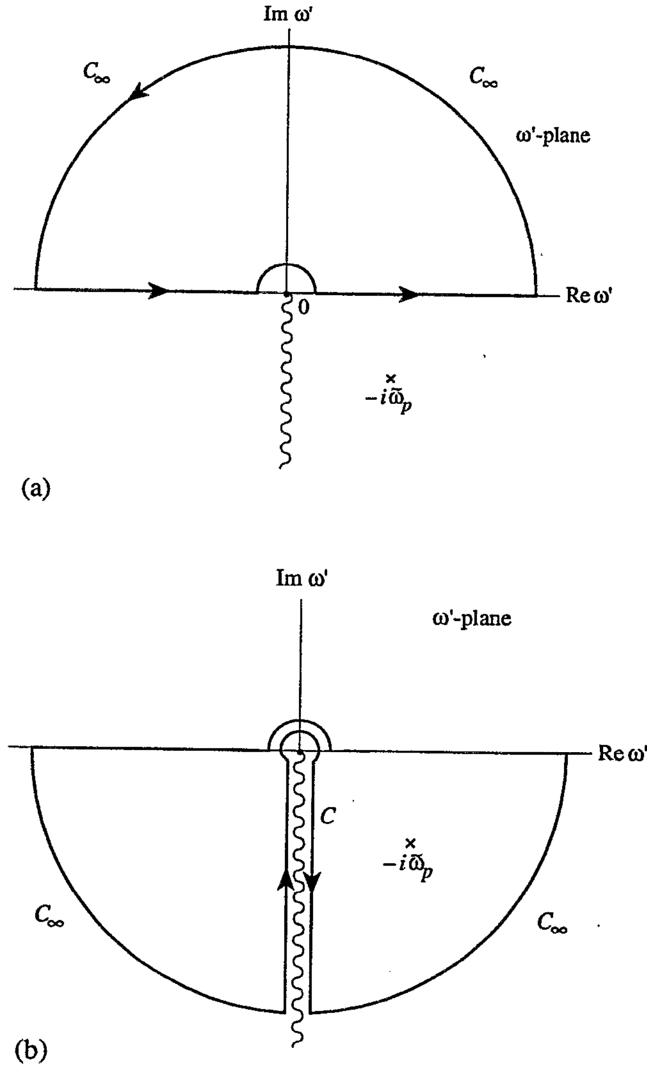

FIG. 1. The contour of integration for the integral $I_3(\rho',t')$ when (a) $t'<0$, and (b) $t'>0$. In (b), part C of the contour encloses both sides of the branch cut in the lower half-plane.

$$I_2(\rho',t')=\frac{i-1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\sqrt{\omega'}}\frac{\omega_p'}{\omega'+i\omega_p'}\times e^{-\rho'\sqrt{\omega'}}e^{-i(\omega't'-\rho'\sqrt{\omega'})}d\omega', \qquad (18)$$

$$I_3(\rho',t')=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\omega'}\frac{\omega_p'}{\omega'+i\omega_p'}\times e^{-\rho'\sqrt{\omega'}}e^{-i(\omega't'-\rho'\sqrt{\omega'})}d\omega'. \qquad (19)$$

In Eqs. (17)–(19), each integrand has a branch point at $\omega'=0$ and a simple pole at $\omega'=-i\omega_p'$ in the lower half-plane. The branch cut is chosen to be along the negative imaginary axis and the path of integration is along the real axis with an indentation about $\omega'=0$ in the upper half-plane, as shown in Fig. 1. Note that

$$I_2(\rho',t') = -\frac{\partial I_3(\rho',t')}{\partial \rho'}; \quad I_1(\rho',t') = 2\frac{\partial I_3(\rho',t')}{\partial t'}, \tag{20}$$

by interchanging the order of integration and differentiation, since $I_1(\rho',t')$, $I_2(\rho',t')$, and $I_3(\rho',t')$ each exists and converges uniformly with respect to t' and $\rho'>0$. Therefore, it is sufficient to evaluate only $I_3(\rho',t')$.

IV. EVALUATION OF $I_3(\rho',t')$

In the following analysis, ω_p' is replaced by a complex quantity $\tilde{\omega}_p$ such that

$$\tilde{\omega}_p = \omega_p' + i\omega_1' = |\tilde{\omega}_p| e^{i\theta}, \tag{21}$$

where $\omega_1'>0$ and $0<\theta<\pi/2$. Then,

$$-i\tilde{\omega}_p = |\tilde{\omega}_p| e^{-i(\pi/2-\theta)} \quad \text{or} \quad \sqrt{-i\tilde{\omega}_p} = e^{-i\pi/4}\sqrt{\tilde{\omega}_p}, \tag{22}$$

where

$$\sqrt{\tilde{\omega}_p} = \sqrt{|\tilde{\omega}_p|}\, e^{i\theta/2}. \tag{23}$$

For $t'<0$, the path of integration may be closed by a large semicircle in the upper half-plane, as shown in Fig. 1(a). It follows that

$$I_3(\rho',t') = 0, \quad t'<0, \tag{24}$$

since the function is holomorphic in the upper half-plane. For $t'>0$, the path of integration may be closed in the lower half-plane, as shown in Fig. 1b, and $I_3(\rho',t')$ can be written as

$$I_3(\rho',t') = I_p(\rho',t') + I_b(\rho',t'), \tag{25}$$

where $I_p(\rho',t')$ is the contribution of the simple pole, namely,

$$I_p(\rho',t') = -e^{i\rho'\sqrt{2\tilde{\omega}_p}} e^{-\tilde{\omega}_p t'}, \tag{26}$$

and $I_b(\rho',t')$ is the contribution of both the branch cut and the branch point:

$$I_b(\rho',t') = -\frac{1}{2\pi}\int_C \frac{1}{\omega'}\frac{\tilde{\omega}_p}{\omega'+i\tilde{\omega}_p} \times e^{-\rho'\sqrt{\omega'}} e^{-i(\omega' t' - \rho'\sqrt{\omega'})} d\omega', \tag{27}$$

where the contour C encloses the branch cut, upward on the left-hand side and downward on the right-hand side, and encircles the branch point with a small circle of radius δ.

In order to simplify the expression for $I_b(\rho',t')$ in Eq. (27), let $\omega' = e^{3i\pi/2}\xi$ on the left-hand side of the branch cut and $\omega' = e^{-i\pi/2}\xi$ on the right-hand side of the branch cut. Then, it follows that

$$I_b(\rho',t') = -\frac{1}{2\pi}\Bigg[\lim_{\delta\to 0^+}\int_{C_\delta}\frac{1}{\omega'}\frac{\tilde{\omega}_p}{\omega'+i\tilde{\omega}_p} e^{-\rho'\sqrt{\omega'}} \times e^{-i(\omega' t'-\rho'\sqrt{\omega'})}d\omega' - \int_0^\infty \frac{d\xi}{\xi}\frac{\tilde{\omega}_p}{-i\xi+i\tilde{\omega}_p} e^{-\xi t'} \times \big(e^{-\rho'\sqrt{\xi}(-1+i)/\sqrt{2}} e^{i\rho'\sqrt{\xi}(-1+i)/\sqrt{2}} - e^{-\rho'\sqrt{\xi}(1-i)/\sqrt{2}} \times e^{i\rho'\sqrt{\xi}(1-i)/\sqrt{2}}\big)\Bigg] = 1 + \frac{1}{\pi}\bar{I}_b(\rho',t'), \tag{28}$$

where

$$\bar{I}_b(\rho',t') = \int_0^\infty \frac{d\xi}{\xi}\frac{\tilde{\omega}_p}{\xi-\tilde{\omega}_p}\sin(\rho'\sqrt{2\xi})e^{-\xi t'}. \tag{29}$$

Consequently,

$$\frac{\partial}{\partial t'}[e^{\tilde{\omega}_p t'}\bar{I}_b(\rho',t')] = -e^{\tilde{\omega}_p t'}\tilde{\omega}_p I(\rho',t'), \tag{30}$$

where

$$I(\rho',t') = \int_0^\infty \frac{d\xi}{\xi}\sin(\rho'\sqrt{2\xi})e^{-\xi t'}. \tag{31}$$

This integral can be evaluated exactly as follows:

$$\frac{\partial I(\rho',t')}{\partial \rho'} = \sqrt{2}\int_0^\infty \frac{e^{-\xi t'}}{\sqrt{\xi}}\cos(\rho'\sqrt{2\xi})d\xi. \tag{32}$$

With

$$\zeta = \sqrt{t'}\left(\sqrt{\xi} - \frac{i\rho'}{\sqrt{2}t'}\right),$$

Eq. (32) becomes

$$\frac{\partial I(\rho',t')}{\partial \rho'} = \frac{2\sqrt{2}}{\sqrt{t'}} e^{-\rho'^2/2t'}\,\mathrm{Re}[\tilde{I}(\rho',t')], \tag{33}$$

where

$$\tilde{I}(\rho',t') = \int_{-i\rho'/\sqrt{2t'}}^\infty e^{-\zeta^2}d\zeta = \frac{\sqrt{\pi}}{2}\,\mathrm{erfc}\left(-\frac{i\rho'}{\sqrt{2t'}}\right). \tag{34}$$

But,

$$\mathrm{erfc}\left(-\frac{i\rho'}{\sqrt{2t'}}\right) = 1 - \mathrm{erf}\left(-\frac{i\rho'}{\sqrt{2t'}}\right), \tag{35}$$

and

$$\mathrm{erf}(w) \equiv \frac{2}{\sqrt{\pi}}\int_0^w e^{-s^2}ds, \tag{36}$$

from Eqs. (7.1.2) and (7.1.1) on p. 297 of Abramowitz and Stegun.[5] For $w=-i\rho'/\sqrt{2t'}$, i.e., pure imaginary, and a change of variable $s=-(i\rho'/\sqrt{2t'})\xi$ in the last integral, it becomes obvious that $\mathrm{erf}(w)$ is also pure imaginary, i.e., $\mathrm{Re}[\mathrm{erf}(w)]=0$. Consequently,

$$\frac{\partial I(\rho',t')}{\partial \rho'} = \sqrt{\frac{2\pi}{t'}}\, e^{-\rho'^2/2t'}. \tag{37}$$

From Eq. (31), $I(\rho'=0,t')=0$. Hence,

$$I(\rho',t') = \sqrt{\frac{2\pi}{t'}}\int_0^{\rho'} e^{-u^2/2t'}du = \pi\,\mathrm{erf}\left(\frac{\rho'}{\sqrt{2t'}}\right). \tag{38}$$

With Eq. (30), it follows directly that

$$\frac{\partial}{\partial t'}[e^{\tilde{\omega}_p t'}\bar{I}_b(\rho',t')] = -e^{\tilde{\omega}_p t'}\tilde{\omega}_p\pi\,\mathrm{erf}\left(\frac{\rho'}{\sqrt{2t'}}\right). \tag{39}$$

By the use of Eq. (29), the initial value of $\bar{I}_b(\rho',t')$ reads

$$\bar{I}_b(\rho',t'=0)=\int_0^\infty \frac{d\xi}{\xi}\frac{\tilde{\omega}_p}{\xi-\tilde{\omega}_p}\ \sin\ (\rho'\sqrt{2\xi}). \tag{40}$$

Let $\sqrt{\xi}=x$. Then,

$$\bar{I}_b(\rho',t'=0)=2\int_0^\infty \frac{\tilde{\omega}_p}{x(x^2-\tilde{\omega}_p)}\ \sin\ (\rho'\sqrt{2}x)dx$$

$$=\frac{\tilde{\omega}_p}{2i}\left\{\int_{-\infty}^{\infty}\frac{e^{i\rho'\sqrt{2}x}}{x(x^2-\tilde{\omega}_p)}\,\mathrm{dx}\right.$$

$$\left.-\int_{-\infty}^{\infty}\frac{e^{-i\rho'\sqrt{2}x}}{x(x^2-\tilde{\omega}_p)}\,dx\right\}, \tag{41}$$

and the path of integration is properly indented about $x=0$. These integrals are elementary and can be evaluated by contour integration in the complex plane, where each integrand has 3 simple poles at 0, $\pm\sqrt{\tilde{\omega}_p}$:

$$\bar{I}_b(\rho',t'=0)=-\pi+\pi e^{i\rho'\sqrt{2\tilde{\omega}_p}}. \tag{42}$$

With Eq. (39), $\bar{I}_b(\rho',t')$ is readily evaluated in terms of a new integral:

$$\bar{I}_b(\rho',t')=\bar{I}_b(\rho',t'=0)e^{-\tilde{\omega}_p t'}$$

$$-\pi\tilde{\omega}_p\int_0^{t'} e^{-\tilde{\omega}_p(t'-\xi)}\ \bar{\mathrm{erf}}\left(\frac{\rho'}{\sqrt{2\xi}}\right)d\xi. \tag{43}$$

If $\tilde{\omega}_p\to\omega_p'$, from Eqs. (25), (26), (28), (42), and (43), it follows that

$$I_3(\rho',t')=\omega_p'\int_0^{t'} e^{-\omega_p'\tau}\ \mathrm{erfc}\left(\frac{\rho'}{\sqrt{2(t'-\tau)}}\right)d\tau, \tag{44}$$

since the result is independent of the position of the branch cut in the lower half-plane.

Next, let $\tau=t'\zeta$. Then, Eq. (44) yields

$$I_3(\rho',t')=\omega_p't'\int_0^1 e^{-(\omega_p't')\zeta}\ \mathrm{erfc}\left(\frac{\rho'}{\sqrt{2t'}}\frac{1}{\sqrt{1-\zeta}}\right)d\zeta, \tag{45}$$

where $\omega_p't'=\omega_p t$ and $\rho'/\sqrt{2t'}=a\rho/\sqrt{2t}$ from Eq. (14).

Now let

$$\Omega=\sqrt{\omega_p't'}\quad\text{and}\quad R=\frac{\rho'}{\sqrt{2t'}}. \tag{46}$$

After integration by parts, Eq. (45) gives

$$I_3(\rho',t')=\mathrm{erfc}(R)-\frac{R}{\sqrt{\pi}}\int_0^1(1-\zeta)^{-3/2}$$

$$\times\exp\left(-\Omega^2\zeta-\frac{R^2}{1-\zeta}\right)d\zeta. \tag{47}$$

With the change of variable $\xi=(1-\zeta)^{-1/2}$,

$$I_3(\rho',t')=\mathrm{erfc}\ (R)-\frac{2R}{\sqrt{\pi}}e^{-\Omega^2}\bar{I}_3(\Omega,R), \tag{48}$$

where

$$\bar{I}_3(\Omega,R)=\int_1^\infty \exp\left(\frac{\Omega^2}{\xi^2}-R^2\xi^2\right)d\xi. \tag{49}$$

After some straightforward algebra,

$$\exp\left(\frac{\Omega^2}{\xi^2}-R^2\xi^2\right)=\frac{1}{2R}\frac{d}{d\xi}\left(e^{-2i\Omega R}\int_0^{R\xi-i\Omega/\xi}e^{-t^2}dt\right.$$

$$\left.+e^{2i\Omega R}\int_0^{R\xi+i\Omega/\xi}e^{-t^2}dt\right),$$

resulting in

$$\bar{I}_3(\Omega,R)=\frac{\sqrt{\pi}}{2R}\ \mathrm{Re}[e^{2i\Omega R}\ \mathrm{erfc}(R+i\Omega)]. \tag{50}$$

With Eqs. (48) and (50),

$$I_3(\rho',t')=\mathrm{erfc}(R)-e^{-R^2}\ \mathrm{Re}[e^{Z^2}\,\mathrm{erfc}(Z)], \tag{51}$$

where

$$Z=R+i\Omega=\frac{\rho'}{\sqrt{2t'}}+i\sqrt{\omega_p't'}. \tag{52}$$

The locus of $Z=Z(t')$ in the complex plane is the hyperbola defined by the equation

$$\mathrm{Re}(Z)\cdot\mathrm{Im}(Z)=\sqrt{\frac{\omega_p'}{2}}\rho', \tag{53}$$

where Re(Z), Im(Z)>0. The minimum distance of this hyperbola from the origin equals

$$|Z|_{\min}=[(2\omega_p')^{1/2}\rho']^{1/2}. \tag{54}$$

If $|Z|_{\min}\gg 1$, i.e.,

$$a\rho\sqrt{2\omega_p}\gg 1, \tag{55}$$

then, by the use of Eq. (7.1.23) of Ref. 5, $I_3(\rho',t')$ can be approximated by the leading term

$$I_3(\rho',t')\sim\mathrm{erfc}\left(\frac{\rho'}{\sqrt{2t'}}\right)$$

$$-\sqrt{\frac{2t'}{\pi}}e^{-\rho'^2/2t'}\frac{\rho'}{\rho'^2+2\omega_p't'^2}, \tag{56}$$

which is valid uniformly in times $0\leqslant t<\infty$.

V. EVALUATION OF $E_z(\rho,t)$

Once $I_3(\rho',t')$ has been obtained, $I_2(\rho',t')$ and $I_1(\rho',t')$ may readily be evaluated with Eqs. (20). Finally, by the use of Eqs. (13), (15), and (16), the following formula has been derived:

$$E_z(\rho,t)=\frac{\mu_0 a h_e I_0}{8\pi t_1}\begin{cases}0, & t<0,\\ \mathscr{E}(\rho,t), & 0<t<2t_1,\\ \mathscr{E}(\rho,t)-\mathscr{E}(\rho,t-2t_1), & t>2t_1,\end{cases} \tag{57}$$

where

$$\mathscr{E}(\rho,t)=\frac{e^{-R^2}}{t\sqrt{2t}}\left(\frac{1}{2R^3}[F(Z)-F(R)]+\frac{\Omega}{R^2}G(Z)-\frac{2\Omega^2}{R}F(Z)\right), \quad (58)$$

$$F(Z)=\mathrm{Re}[e^{Z^2}\,\mathrm{erfc}(Z)];\quad G(Z)=\mathrm{Im}[e^{Z^2}\,\mathrm{erfc}(Z)], \quad (59)$$

$$Z=R+i\Omega,\quad R=\frac{a\rho}{\sqrt{2t}},\quad \Omega=\sqrt{\omega_p t}. \quad (60)$$

It is worth noting that the complete field includes terms proportional to $1/R^3$, $G(Z)\cdot 1/R^2$, and $F(Z)\cdot 1/R$ corresponding to the near, intermediate, and far fields, respectively. This is a consequence of the use of the parameter $R=R(t)$. For fixed time, $0<t<2t_1$, the spatial dependence of the field changes from $1/\rho^3$ to e^{-R^2}/ρ^2 as the pulse moves from the near $(R\ll 1)$ to the far field $(R\gg 1)$.

$\mathscr{E}(\rho,t)$ may be written as the sum of two terms:

$$\mathscr{E}(\rho,t)=\mathscr{E}_0(\rho,t)+\mathscr{E}_1(\rho,t), \quad (61)$$

where

$$\mathscr{E}_0(\rho,t)=-\frac{e^{-R^2}}{t\sqrt{2\pi t}}\left(2+\frac{1}{R^2}+\frac{\sqrt{\pi}}{2R^3}F(R)\right), \quad (62)$$

$$\mathscr{E}_1(\rho,t)=\frac{e^{-R^2}}{t\sqrt{2t}}\left[\frac{1}{2R^3}F(Z)+\frac{1}{R^2}\left(\Omega G(Z)+\frac{1}{\sqrt{\pi}}\right)-\frac{2\Omega^2}{R}F(Z)+\frac{2}{\sqrt{\pi}}\right], \quad (63)$$

and $\mathrm{l.i.m.}_{\omega_p\to+\infty}\mathscr{E}_1(\rho,t)=0$, while $\mathscr{E}_0(\rho,t)$ is independent of ω_p. Therefore, $\mathscr{E}_0(\rho,t)$ represents the response to the ideal rectangular-excitation pulse.

In the limit $2t_1\to 0^+$, the response to the discontinuous exponential-excitation pulse [Eq. (7)] can be evaluated by expansion of $\mathscr{E}(\rho,t-2t_1)$ in Taylor series about $t_1=0$. Then,

$$E_z(\rho,t)=\frac{\mu_0 a h_e I_0}{4\pi}\frac{\partial\mathscr{E}(\rho,t)}{\partial t},\quad t>0, \quad (64)$$

i.e., it is the time derivative of the response to the continuous exponential pulse.

An interesting case involves the limits $\omega_p\to+\infty$ and $2t_1\to 0^+$ when these are taken consecutively. With Eqs. (61)–(63), $E_z(\rho,t)$ reduces to

$$E_z(\rho,t)=\frac{\mu_0 a(2h_e)I_0}{8\pi^2 t^2}\sqrt{\frac{2\pi}{t}}\left(1-\frac{a^2\rho^2}{2t}\right)e^{-a^2\rho^2/2t}, \quad (65)$$

in accord with Eq. (31) of Ref. 4, which is concerned with the so-called "late-time" approximation of the "exact" impulse response of a short electric dipole. Emphasis should be placed on the derivation of Eq. (65) by the use of the low-frequency approximation. This result clearly proves that the "late-time" response to the nonrealistic delta-function excitation corresponds to its low-frequency part of the spectrum and, therefore, can be attributed to a realistic low-frequency pulse as well.

Finally, when $|Z|\gg 1$, Eq. (63) can be simplified by the use of the asymptotic expansion [Ref. 5, Eq. (7.1.23)]:

$$e^{Z^2}\,\mathrm{erfc}(Z)=\frac{1}{\sqrt{\pi}\,Z}\left[1-\frac{1}{2Z^2}+O\left(\frac{1}{Z^4}\right)\right], \quad (66)$$

valid uniformly in time if condition (55) is satisfied. In particular, when $\omega_p t=\Omega^2\gg R^2=a^2\rho^2/2t$, Eq. (63) reduces to

$$\mathscr{E}_1(\rho,t)\sim-\sqrt{\frac{2}{\pi t}}\,\frac{1}{\omega_p t^2}\left(1-\frac{a^2\rho^2}{2t}\right)e^{-a^2\rho^2/2t}, \quad (67)$$

which represents the first-order correction due to the nonzero rise time and agrees with previous results.[3,6] This agreement is consistent with the fact that the major contribution to integration in the integrals involved in the transient response comes from the vicinity of the fixed point $\omega'=0$ in the frequency domain and is, therefore, determined by the local analytic behavior of the respective Fourier transform.

VI. CONCLUSIONS

The analytically evaluated electric field generated by an electrically short dipole in sea water excited by a pulse with a finite, nonzero rise and decay time reveals a complicated behavior. This field consists of two transients, each one of them being the sum of two terms. The first term represents the response to the rectangular pulse with zero rise and decay time, while the second term is the correction accounting for the step discontinuity in the first derivative of the excitation pulse. Each transient contains the complete field, and shifts successively from the form of the original pulse to the spatial and time derivatives.

The results obtained may be applied to remote sensing in sea water, when low-frequency pulses are used. Since a similar analysis can be followed for any highly dissipative medium, the conclusions derived are expected to describe pulse propagation in the human body as well, provided that a proper carrier frequency is chosen for modulation of the low-frequency signal.

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