Math 241

February 19, 2024

Exam 1 - Time: 50 minutes

Dr. Ebrahimian

Name:

UID:

- Calculators, formula sheets, notes, and electronic devices, are not allowed. Only writing utensils, sharpeners and erasers are allowed.
- Show your work completely and clearly.
- Answer each problem on the allocated space only.
- Please **turn off all electronic devices**. No electronic device may be on or visible at any time during the exam.
- There are a total of 101 points to be earned. One extra point thanks to the primeness of 101!
- Unless you are asked to, you do not need to simplify your final numerical answers.
- Unless specified, you need to evaluate all integrals.
- Quantities such as $\ln 1$, $\sin(\pi/4)$, etc. must be replaced by their numerical values.
- **Please do not start before you are told to do so.**
- All pages are double sided.

1. Consider the vectors $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and the point $A = (1, 2, 1)$.

1a. (15 pts) Find an equation of the plane that passes through A and is parallel to both $\mathbf{u}$ and $\mathbf{v}$.

1b. (10 pts) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

2. Consider the points $A = (1, 2, -1), B = (0, 1, 2)$, and $C = (2, 3, 1)$

2a. (15 pts) Find the distance from A to the line through B and C.

2b. (10 pts) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

3. The velocity of a particle in space at time t is given by $\mathbf{v}(t) = (t^2 - 2)\mathbf{i} - 2t\mathbf{j} - 2t\mathbf{k}$.

3a. (10 pts) Given the initial position $\mathbf{r}(0) = (0, 0, 1)$, find the position of the particle at time $t = 1$.

3b. (15 pts) Find the length of the curve traced by this particle from $t = 0$ to $t = 1$.

4a. (13 pts) Determine if the parametrization $\mathbf{r}(t) = (t - \sin t)\mathbf{i} - (\cos t)\mathbf{j} - \mathbf{k}$ with $0 \leq t \leq 4\pi$, is smooth, piece-wise smooth or neither.

4b. (13 pts) Find the curvature of this curve at $t = \frac{\pi}{2}$. Simplify your answer.

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