Math 241

March 11, 2024

Exam 2 - Time: 50 minutes

Dr. Ebrahimian

Please clearly print your name and UID in the appropriate space.

Name:

UID:

- Calculators, formula sheets, notes, and electronic devices, are not allowed. Only writing utensils, sharpeners and erasers are allowed.
- Show your work completely and clearly.
- Answer each problem on the allocated space only.
- Please **turn off all electronic devices**. No electronic device may be on or visible at any time during the exam.
- There are a total of 101 points to be earned. One extra point thanks to the primeness of 101!
- Unless you are asked to, you do not need to simplify your final numerical answers.
- Unless specified, you need to evaluate all integrals.
- Quantities such as $\ln 1$, $\sin(\pi/4)$, etc. must be replaced by their numerical values.
- **Please do not start before you are told to do so.**
- All pages are double sided.

1a. (15 pts) Approximate $\sqrt{(2.01)^3 + (0.98)^3}$ using the tangent plane approximation. Simplify your answer.

1b. (10 pts) Find the direction, as a unit vector, in which the function $x^2y + y^2e^x$ decreases most rapidly at $(0, 1)$. What is this minimum directional derivative?

2. Consider the function $f(x,y) = \dfrac{y^6}{x^4+y^6}$.

2a. (13 pts) Evaluate the limit or show it does not exist: $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

2b. (12 pts) Given $x = u + v^2$, and $y = u - v$, use Chain Rule to find $\dfrac{\partial f}{\partial v}$. Your answer may be in terms of u, v, x, y. You **must** use the Chain Rule for this problem.

3. Consider the function $f(x, y) = x^2 - 4xy + y^3$.

3a. (11 pts) Determine all critical points of $f(x, y)$.

3b. (15 pts) Determine if each critical point is a local maximum, a local minimum or a saddle point.

4. (25 pts) Using Lagrange Multiplier's method find the minimum and maximum distance from the point $(5, 0)$ to the points on the ellipse $4x^2 + 9y^2 = 36$. Assume these minimum and maximum values exist.

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