Math 241 Spring 2012 Final Exam

- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets.
- No calculators are permitted.
- One page of notes is permitted.
- Do not evaluate integrals or simplify answers unless indicated.

Please put problem 1 on answer sheet 1

1. Let ℓ be the line with symmetric equations

$$\frac{x-1}{3} = \frac{y-2}{12} \ , \ z = 5$$

   (a) Show that the line containing the points $(5, 7, 9)$ and $(6, 11, 9)$ is parallel to ℓ. [5 pts]

   (b) Show that the point $(1, 4, 5)$ is not on ℓ. [5 pts]

   (c) Find the distance between ℓ and the origin. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Show that if $\bar{F}''(t)$ exists for all t then [10 pts]

$$\frac{d}{dt}\left[\bar{F}(t) \times \bar{F}'(t)\right] = \bar{F}(t) \times \bar{F}''(t)$$

   (b) Let C be parametrized by $\bar{r}(t) = e^t \cos t\, \hat{\imath} + e^t \sin t\, \hat{\jmath}$ for $0 \le t \le 3\pi$. Find the length of C. [20 pts]

Please put problem 3 on answer sheet 3

3. Find the curvature κ of the curve at $t = 0$ parametrized by [20 pts]

$$\bar{r}(t) = \frac{1}{3}(1+t)^{3/2}\,\hat{\imath} + \frac{1}{3}(1-t)^{3/2}\,\hat{\jmath} + \frac{\sqrt{2}}{2}\,\hat{k}$$

Please put problem 4 on answer sheet 4

4. Find all critical points of the function $f(x, y) = x^3 + 6xy + 3y^2 - 9x$ and classify each one as a relative maximum, relative minimum, or saddle point. [20 pts]

Turn Over!

Please put problem 5 on answer sheet 5

5. Let $f(x, y, z) = ye^{y-x} - z^2$.

   (a) Consider the level surface S given by $f(x, y, z) = 0$. Find the equation of the plane tangent to S at the point $(1, 1, -1)$. Write your answer in the form $ax + by + cz = d$. [10 pts]

   (b) Find a unit vector in the direction in which f decreases most rapidly at $(1, 1, -1)$. [5 pts]

   (c) What is the maximal directional derivative of f at the point $(1, 1, -1)$? [5 pts]

Please put problem 6 on answer sheet 6

6. (a) Evaluate and simplify the integral $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(x^2)\ dx\ dy$ [15 pts]

   (b) Express the volume of the solid region D bounded above by the paraboloid $z = 3 - x^2 - y^2$ and below by the plane $z = -6$ as an iterated integral. Evaluate and simplify. [15 pts]

Please put problem 7 on answer sheet 7

7. (a) Find $\int_C (2xy\,\hat{\imath} + x^2\,\hat{\jmath} + 1\,\hat{k}) \cdot d\bar{r}$ where C is any curve from $(1, 1, 1)$ to $(5, 12, 12)$. [10 pts]

   (b) Let Σ be the part of $z = x^2 + y^2$ below $z = 9$ and having $y \leq 0$. Let C be the boundary (edge) of Σ with counterclockwise orientation when viewed from above. Define $\bar{F}(x, y, z) = x\,\hat{\imath} + yz\,\hat{\jmath} + y\,\hat{k}$. Apply Stokes' Theorem to the integral $\int_C \bar{F} \cdot d\bar{r}$. until you have an iterated double integral and then stop. [20 pts]

Please put problem 8 on answer sheet 8

8. (a) Let Σ be the part of the cylinder $x^2 + z^2 = 9$ above the xy-plane and between $y = -1$ and $y = 2$ with outwards orientation. Define $f(x, y, z) = yz$. Evaluate the surface integral $\iint_\Sigma f\ dS$. Proceed until you have an iterated double integral and then stop. [20 pts]

   (b) Evaluate the integral $\iint_\Sigma \left(2x\,\hat{\imath} + 4y\,\hat{\jmath} - z\,\hat{k}\right) \cdot \bar{n}\ dS$ where Σ is the sphere of radius 3 centered at the origin with inwards orientation. [10 pts]

Welcome to the End of the Exam