*This test has **10 question(s)** for a total of 200 points.*

Instructions:

- Number the answer sheets from 1 to 10. Write your **name and section number on each answer** sheet (write and sign the Honor Pledge on page 1 only).
- **Answer question #1 on Answer sheet #1, question #2 on Answer sheet #2, etc.**
- This is a closed book exam and calculators and electronic devices are not permitted.
- You may continue your answers on the back of the answer sheets, but please be sure to indicate that there is work on the back.
- Show all work in order to receive full credit. Answers that are unjustified will receive no credit.
- Simplify all answers unless otherwise indicated.

[20 P.] 1. Consider the line L with symmetric equations

$$\frac{x+1}{2} = \frac{y+3}{3} = -z.$$

Determine an equation for the plane containing the line L and passing through $P = (1, 1, -1)$. Express your answer in the form $ax + by + cz = d$.

[10 P.] 2. (a) Find the point on the plane $x + 2y + 5z = 10$ closest to the origin.

[10 P.] (b) Let $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 3\mathbf{j} - 4\mathbf{k}$. Write $\mathbf{b} = \mathbf{u} + \mathbf{v}$, where $\mathbf{u}$ is parallel to $\mathbf{a}$ and $\mathbf{v}$ is perpendicular to $\mathbf{a}$.

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3. Consider an object with position given by $\mathbf{r}(t) = 3\cos 5t\mathbf{i} + 3\sin 5t\mathbf{j} + 2t\mathbf{k}$, where $0 \le t \le 2\pi$.

[6 P.] (a) Compute the velocity and acceleration of the object at any time t.

[4 P.] (b) Is the velocity orthogonal to the acceleration? Justify your answer.

[10 P.] (c) Compute the length of the curve C parametrized by $\mathbf{r}$.

[10 P.] 4. (a) Compute the gradient of $f(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 4)$.

[10 P.] (b) Approximate $\sqrt{(2.9)^2 + (4.1)^2}$.

[20 P.] 5. Use Lagrange multipliers to find the maximum and minimum of $f(x, y) = xy$ subject to the constraint $x^2 + 2y^2 = 1$.

[20 P.] 6. Let D be region in the xy-plane bounded by the curves $y = x^{1/3}$ and $y = x^3$ where $x \ge 0$ and $y \ge 0$. Compute the integral

$$\int\int_D xy\, dA.$$

** TURN OVER**

20 P. 7. Evaluate $\int\int_{\Sigma} \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z) = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and where Σ is the portion of the cylinder $y^2+z^2=4$ between the planes $x=-1$ and $x=3$ oriented by outward directed normal vector.

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20 P. 8. Compute the surface area of the portion of the plane $x+y+z=5$ lying above the unit disk $x^2+y^2\leq 1$.

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20 P. 9. Use spherical coordinates to compute the triple integral

$$\int\int\int_D z^2\,dV$$

where D is the solid region $D = \{(x,y,z)|4\leq x^2+y^2+z^2\leq 9\}$.

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10. Let Σ be the portion of the sphere $x^2+y^2+z^2=2$ above the plane $z=1$ oriented by outward directed normal vector. Let C be the boundary of Σ oriented counterclockwise as viewed from above. Let $I = \int_C y\,dx + xy\,dy + 2xyz\,dz$

12 P. (a) Evaluate I directly using a parametrization of C.

8 P. (b) Use Stokes' Theorem to express I as a double iterated integral over some region R of the xy-plane. Do not evaluate this integral.
