1. (a) Reparametrize the following integral as vertically simple and then evaluate. [15 pts]
**This is the only integral you need to evaluate.**

$$\int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} x\,dx\,dy$$

**Solution:** Picture omitted. We have:

$$\begin{aligned}\int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} x\,dx\,dy &= \int_0^2 \int_{x^2}^{2x} x\,dy\,dx \\ &= \int_0^2 xy\Big|_{x^2}^{2x}\,dx \\ &= \int_0^2 x(2x) - x(x^2)\,dx \\ &= \int_0^2 2x^2 - x^3\,dx \\ &= \frac{2}{3}x^3 - \frac{1}{4}x^4\Big|_0^2 \\ &= \frac{2}{3}(2)^3 - \frac{1}{4}(2)^4\end{aligned}$$

(b) Parametrize the part of the cylinder $x^2 + z^2 = 9$ between $y = 0$ and $y = 5$. [5 pts]
**Solution:** One possibility is:

$$\bar{r}(y,\theta) = 3\cos\theta\,\hat{\imath} + y\,\hat{\jmath} + 3\sin\theta\,\hat{k}$$
$$0 \le y \le 5$$
$$0 \le \theta \le 2\pi$$

2. (a) Let $R$ be the region between the functions $y = x^2$ and $y = 8 - x^2$. Set up the iterated double integral in rectangular coordinates for $\iint_R y\,dA$. [8 pts]

**Do not evaluate.**

**Solution:** The functions meet where $x^2 = 8 - x^2$ or $2x^2 = 8$ or $x = \pm 2$ and so we get

$$\int_{-2}^{2} \int_{x^2}^{8-x^2} y\,dy\,dx$$

(b) Let $R$ be the region inside the circle $r = 4$ and to the right of the line $x = 2$. Set up the iterated double integral in polar coordinates for $\iint_R y\,dA$. [12 pts]

**Do not evaluate.**

**Solution:** The line has polar equation $r\cos\theta = 2$ or $r = 2\sec\theta$. This meets the circle where $2\sec\theta = 4$ or $\cos\theta = \frac{1}{2}$ or $\theta = \pm\frac{\pi}{3}$ so we get

$$\int_{-\pi/3}^{\pi/3} \int_{2\sec\theta}^{4} r\sin\theta\, r\,dr\,d\theta$$

3. Let $D$ be the solid inside the cylinder $(x-2)^2+y^2=4$ and between the planes $z=1$ and $z=7+x$. [20 pts] Write down the iterated triple integral in cylindrical coordinates for the volume of $D$.
**Do not evaluate.**

**Solution:** The cylinder in polar is $r=4\cos\theta$ for $-\pi/2 \leq \theta \leq \pi/2$ and so we have:

$$\iiint_D 1\,dV = \int_{-\pi/2}^{\pi/2}\int_0^{4\cos\theta}\int_1^{7+r\cos\theta} r\,dz\,dr\,d\theta$$

4. Let $D$ be the solid outside the cylinder $x^2 + y^2 = 4$ and inside the sphere $x^2 + y^2 + z^2 = 16$. If the mass of $D$ at a point is given by the function $f(x, y, z) = x^2z^2$, write down the iterated triple integral in spherical coordinates for the mass of $D$. [20 pts]
**Do not evaluate.**

**Solution:** The cylinder has equation $\rho^2 \sin^2 \phi = 4$ or $\rho \sin \phi = 2$ or $\rho = 2 \csc \phi$ and the sphere has equation $\rho = 4$. They meet where $4 \sin \phi = 2$ or $\sin \phi = \frac{1}{2}$ or $\phi = \pi/6$ and $\phi = 5\pi/6$. Therefore:

$$\iiint_D x^2 z^2 \, dV = \int_0^{2\pi} \int_{\pi/6}^{5\pi/6} \int_{2\csc\phi}^{4} (\rho \sin\phi \cos\theta)^2 (\rho\cos\phi)^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

5. Let $R$ be the region in the first quadrant bounded by the lines $y = x$, $y = 3x$, $y = \frac{1}{x}$ and $y = \frac{5}{x}$. [20 pts]
Perform a change of variables to rewrite $\iint_R xy\,dA$ as an iterated integral over a rectangle in the $uv$-plane.
**Do not evaluate.**

**Solution:** The lines are $y/x = 1$, $y/x = 3$, $xy = 1$ and $xy = 5$ so we set $u = y/x$ and $v = xy$ so that $S$ is the rectangle bounded by the lines $u = 1$, $u = 3$, $v = 1$ and $v = 5$.

We then have integrand $xy = v$ and Jacobian

$$J(x, y) = 1 \div J(u, v) = \det \begin{bmatrix} -y/x^2 & 1/x \\ y & x \end{bmatrix} = 1 \div (-2y/x) = 1 \div (-2u) = -\frac{1}{2}u$$

so then

$$\iint_R xy\,dA = \iint_S v \left| -\frac{1}{2}u \right| dA = \int_1^3 \int_1^5 \frac{1}{2}uv\,dv\,du$$