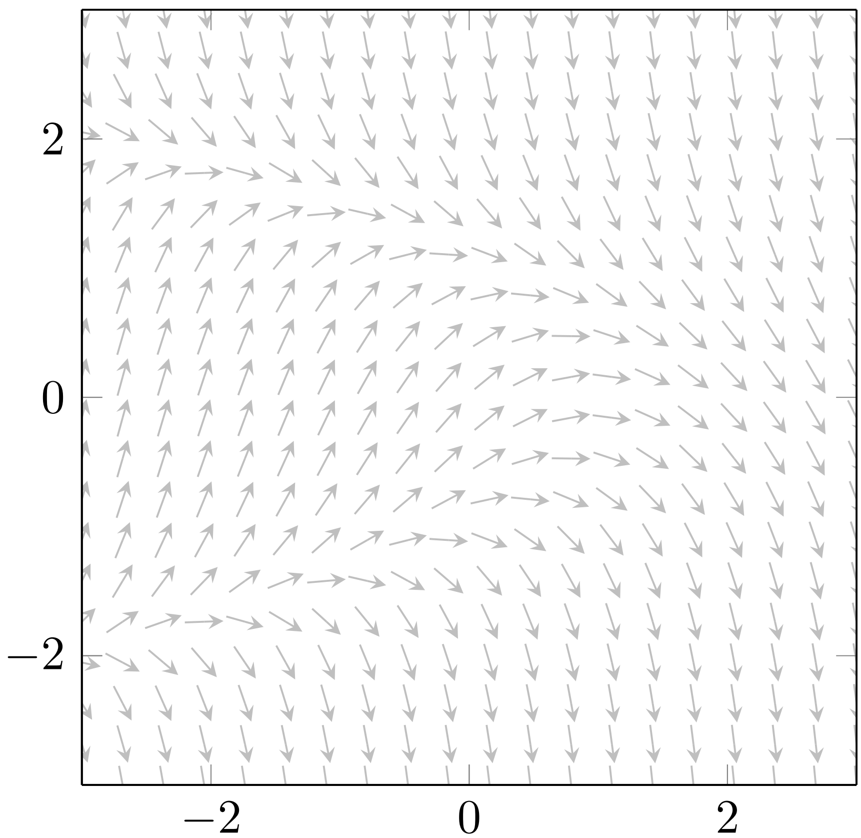
Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

1. Here is the direction field for the differential equation

$$\frac{dy}{dt} = 1 - t - y^2$$

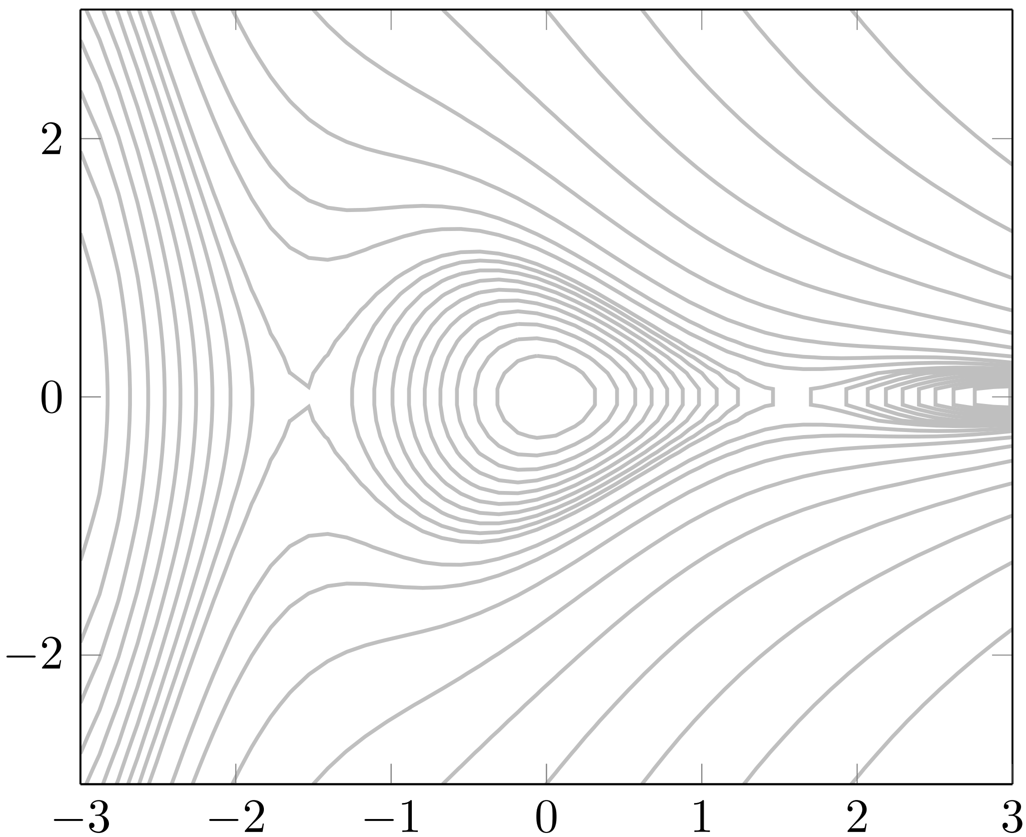


(a) On the field draw the specific solution which satisfies $y(0) = 0$. For this specific solution what (approximately) would be the local minimum? [5 pts]

(b) Consider the initial value $y(-1) = \alpha$. For which α would the solution $y(t)$ be increasing at $t = -1$? [5 pts]

2. Here is the contour plot corresponding to the differential equation

$$\frac{dy}{dx} = -\frac{y^2 e^x + 2\sin x \cos x}{2ye^x}$$



(a) On the contour plot draw the specific solution which satisfies $y(0) = 1$. [3 pts]

(b) On the contour plot draw the specific solution which satisfies $y(-2) = 1$. [3 pts]

(c) Give an example of a pair (t_I, y_I) for which the specific solution satisfying $y(t_I) = y_I$ appears to be an increasing function. [4 pts]

3. Explicitly solve the separable initial value problem. You may assume $t > 0$. [20 pts]

$$t \frac{dy}{dt} = \frac{1}{2y-2} \text{ with } y(e^2) = 3$$

4. Consider the autonomous differential equation

$$y' = (y-2)^2(y-7)$$

   (a) Draw a phase-line portrait. [6 pts]

   (b) Draw a family of solutions for this DE. [8 pts]

   (c) For each constant solution indicate the stability. [3 pts]

   (d) What must be true of α if the specific solution with $y(0) = \alpha$ has $\lim\limits_{t\to\infty} y(t) = 7$? [3 pts]

5. (a) A 50 L tank initially contains 2 L of salt water at a concentration of 5 g/L. Salt water at a concentration of 4 g/L is being pumped in at 2 L/min and the mixture is leaving at 1 L/min. Write down **BUT DO NOT SOLVE** the initial value problem whose solution gives the amount of salt at time t. [10 pts]

   (b) Given the initial value problem [10 pts]

$$\frac{dy}{dt} = ty + t \text{ with } y(1) = 0$$

   Use Euler's Method with step size $h = 0.5$ and two steps to approximate $y(2)$. Make sure your work is neat and your approximations for $y(1.5)$ and $y(2)$ are clearly labeled. Simplify.

6. Consider the initial value problem

$$y + (2x+2)\frac{dy}{dx} = 0 \text{ with } y(2) = 3$$

   (a) Show that this DE is not exact. [5 pts]

   (b) Use the integrating factor $\mu = y$ to solve the IVP implicitly. [15 pts]