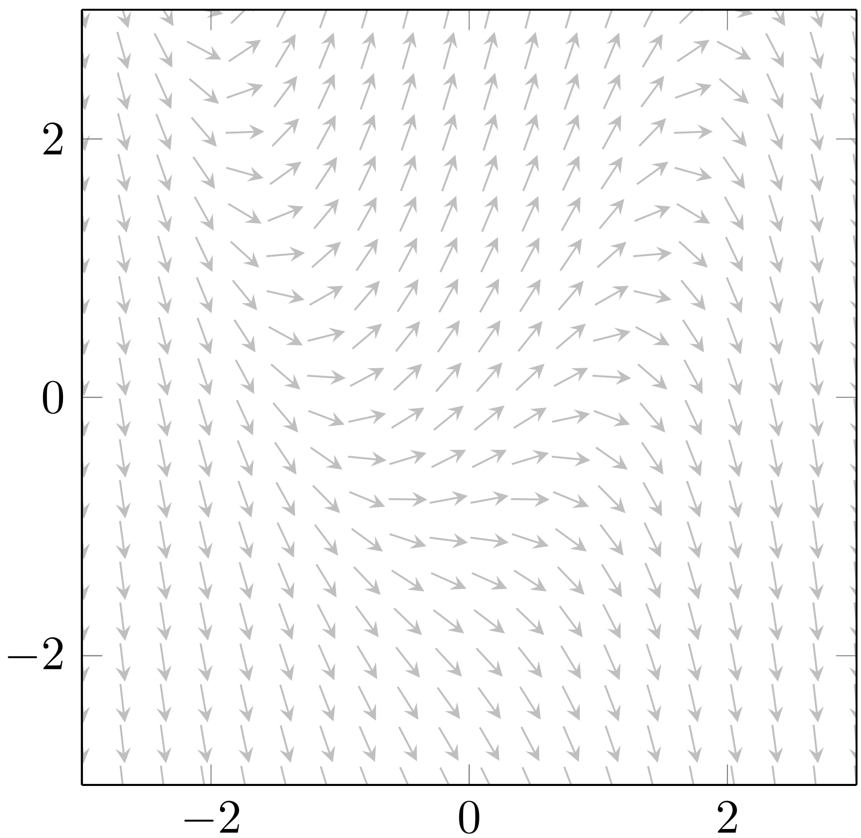
Math 246 Exam 1 Sample 2 **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

1. Here is the direction field for the differential equation

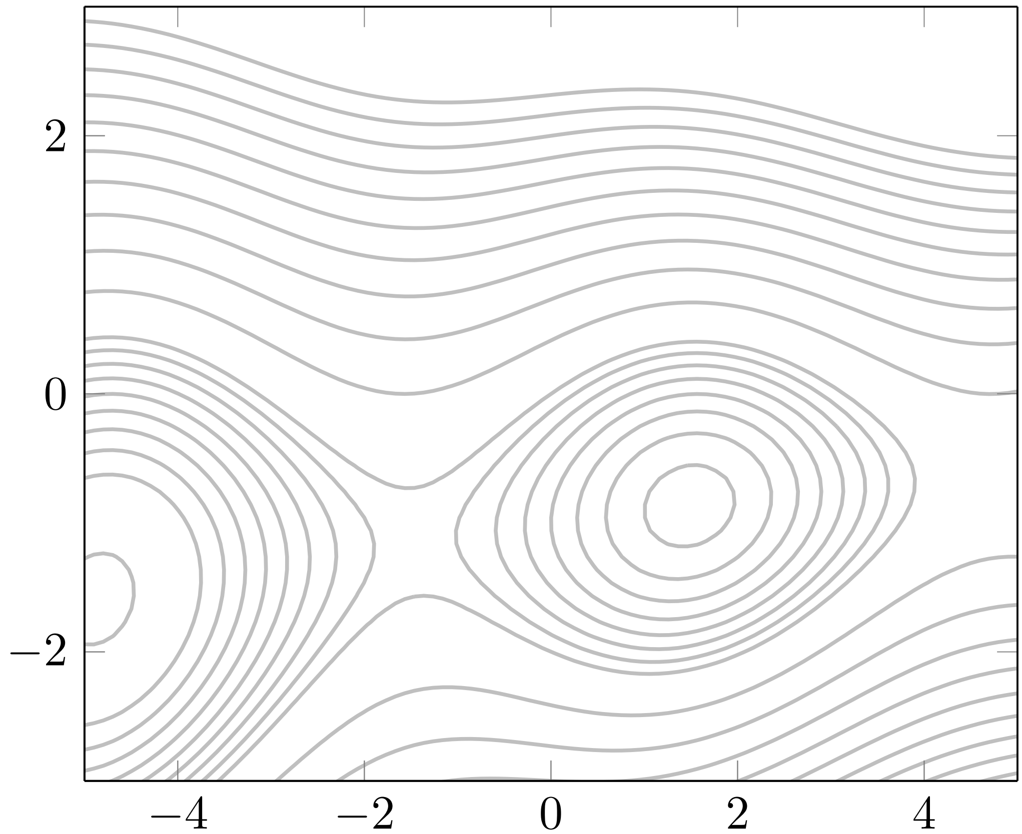
$$\frac{dy}{dt} = 1 - t^2 + y$$



(a) On the field draw the specific solution which satisfies $y(1) = 1$. For this specific solution what (approximately) would be the local minimum? [5 pts]

(b) Consider the initial value $y(0) = \alpha$. For which α would the solution $y(t)$ seem to have a relative minimum? [5 pts]

2. Suppose the implicit solutions to a certain differential equation have the following contour map:



(a) On the contour plot draw the specific solution which satisfies $y(0) = 1$. [3 pts]

(b) On the contour plot draw the specific solution which satisfies $y(0) = -1$. [3 pts]

(c) Give an example of a pair (t_I, y_I) for which the specific solution satisfying $y(t_I) = y_I$ has appears to have a relative minimum and an interval existance which is not $(-\infty, \infty)$. [4 pts]

3. Explicitly solve the initial value problem. [15 pts]

$$y' - 3y = 2t \text{ with } y(1) = 2$$

4. Consider the autonomous differential equation

$$y' = y(y-3)^2(y-6)^2$$

   (a) Draw a phase-line portrait. [5 pts]

   (b) Draw a family of solutions for this DE. [5 pts]

   (c) For each constant solution indicate the stability. [3 pts]

   (d) What must be true of α if the specific solution with $y(0) = \alpha$ has $\lim_{t\to\infty} y(t)$ unbounded? [2 pts]

5. The population of rats in a small town is growing at a rate of 2.5%. If pest control manages to kill 30 rats per day and there were 1000 rats initially, how long will it take until there are only 500 rats remaining? [20 pts]

6. Given the initial value problem [10 pts]

$$\frac{dy}{dt} = \frac{t}{y} + y \text{ with } y(1) = 2$$

   Use Euler's Method with step size $h = 0.5$ and two steps to approximate $y(2)$. Make sure your work is neat and your approximations for $y(1.5)$ and $y(2)$ are clearly labeled. Simplify.

7. Consider the initial value problem

$$y + (2x+2)\frac{dy}{dx} = 0 \text{ with } y(2) = 3$$

   (a) Show that this DE is not exact. [5 pts]

   (b) Use the integrating factor $\mu = y$ to solve the IVP implicitly. [15 pts]