MATH 246: Exam 2 Sample 1

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1. Show that if the Wronskian of Y_1 and Y_2 is nonzero then so is the Wronskian of $2Y_1$ and $Y_1 + Y_2$.

2. Calculate the determinant of the matrix and simplify

$$\det \begin{bmatrix} 1 & 0 & 4 \\ -2 & 1 & 8 \\ 2 & 3 & 3 \end{bmatrix}$$

3. Write down a differential equation for which $Y_P(t) = (At^2 + Bt)\cos(4t) + (Ct^2 + Dt)\sin(4t)$ could be the guess for the Method of Undetermined Coefficients.

4. Write down the undetermined $Y_p(t)$ for $y'' - 2y' - 3y = te^{3t}$ using the Method of Undetermined Coefficients. Do not plug in or solve for the constants.

5. Find the general solution to the differential equation $y'' + 9y' + 20y = 0$.

6. Give an example of a differential equation for which each of the following pairs could be a pair of fundamental solutions:

   (a) $Y_1(t) = e^{2t}$ and $Y_2(t) = e^{-7t}$

   (b) $Y_1(t) = \sin(t)$ and $Y_2(t) = \cos(t)$

   (c) $Y_1(t) = 1$ and $Y_2(t) = t$.

7. The functions $Y_1(t) = t$ and $Y_2(t) = te^t$ form a fundamental pair of solutions to the homogeneous differential equation $t^2y'' - t(t+2)y' + (t+2)y = 0$. Do not prove this. Use Variation of Parameters to find some solution $Y_P(t)$ to the nonhomogeneous differential equation:

$$t^2y'' - t(t+2)y' + (t+2)y = 2t^3$$

8. A weight of 0.5kg stretches a spring by 0.49m. The spring-mass system is submerged in delicious melted butter with a damping coefficient of $\gamma = 4$. The spring is then lowered by an additional 1.0m and released with velocity 0. There is no external force. Find the function which gives the location of the weight at time t.
   Note: I have designed this to work out nicely. If it's not working out nicely then you probably got some butter in your calculations.

9. Solve the initial value problem

$$y'' - 6y' + 9y = 2t + 4 \text{ with } y(0) = 3, y'(0) = -2$$

   Note: I have not designed these coefficients to work out nicely but it's just a bit of fraction work.

10. Start solving $y'' - 2y' + y = e^{3t}$ with $y(0) = 1$, $y'(0) = -1$ using Laplace transforms. Proceed until you have $\mathcal{L}[y] = ...$ on the left side, then stop. You do not need to do partial fractions or simplify the right side.

11. Find y so that $\mathcal{L}[y] = \frac{s+5}{(s-2)^2+9}$