MATH 246: Exam 2 Sample 2

1. Suppose a homogeneous second order differential equation has fundamental pair $\{t, t^3\}$. Solve the IVP with $y(2) = 1$ and $y'(2) = -3$.

2. A 0.2kg weight stretches a spring 0.1m. The system is submerged in oil with damping coefficient $\gamma = 3$. The weight is then lowered by 0.2m and released with a downward velocity of 1m/s. There is no external force.

   (a) Find the spring coefficient k.

   (b) Write down **but do not solve** the initial value problem corresponding to this situation.

   (c) Is this system underdamped, critically damped or overdamped? Show the associated calculation.

   (d) Sketch a reasonable graph of the solution.

3. Write down the general solution to the differential equation $D^5y + 4D^3y = 0$.

4. For the differential equation $y'' - 6y' + 9y = (t^2 + 3)e^{3t}$ write down the undetermined $Y_p(t)$ which you would use in the Method of Undetermined Coefficients. Do not go further.

5. Use the Method of Undetermined Coefficients to find a specific solution $Y_p(t)$ to the differential equation
$$y'' + 5y' - 3y = 3t + 2$$

6. Use Variation of Parameters to find a particular solution to the differential equation
$$t^2y'' + 2ty' - 2y = t^2$$
The homogeneous version has fundamental pair $\{t, t^{-2}\}$. Then write down the general solution.

7. Use the definition (not the table) to calculate $\mathcal{L}[3]$.

8. Use Laplace Transforms to solve the initial value problem
$$y'' - 4y' + 13y = 0 \text{ with } y(0) = 0 \text{ and } y'(0) = -1$$

9. Define the function:
$$f(t) = \begin{cases} 0 & \text{for } t < 7 \\ (t-7)^2 & \text{for } t \geq 7 \end{cases}$$
Solve the initial value problem:
$$y' - y = f(t) \quad \text{with} \quad y(0) = 2$$