MATH 246 Groupwork 2.2   **Name:** _____

1. Check that each pair given form a fundamental pair of solutions to the differential equation. Note that you must check both that they're solutions and that they form a fundamental pair!

   (a) DE: $y'' - 3y' - 10y = 0$
   Pair: $Y_1(t) = e^{-2t}$ and $Y_2(t) = e^{5t}$

   (b) DE: $t^2 y'' + 4ty' = 0$
   Pair: $Y_1(t) = 1$ and $Y_2(t) = t^{-3}$

2. Solve the following initial value problem and give the interval of existence.
   Hint: Use the appropriate pair from the previous problem:

   $$t^2 y'' + 4ty' = 0 \text{ with } y(-1) = 3, y'(-1) = 5$$

3. What's wrong with the following argument:
   Given the differential equation $ty'' - t^2 y' + 5y = 0$, the Wronskian of the functions $Y_1(t) = t^2$
   and $Y_2(t) = t^3$ is

   $$W\left[t^2, t^3\right] = \begin{vmatrix} t^2 & t^3 \\ 2t & 3t^2 \end{vmatrix} = 3t^4 - 2t^4 = t^4 \neq 0$$

   and hence Y_1 and Y_2 form a fundamental pair of solutions to the DE.