MATH 246 Homework 1.5
Justin Wyss-Gallifent

Directions:

- Work should be done neatly on these sheets!
- Enough work must be shown so that the steps you are taking is clear.

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1. Consider the following autonomous differential equation:

$$y' = (y+3)(y-7)^2(y-10)$$

   (a) Draw a phase-line portrait for this DE.

   (b) Sketch a reasonable family of solutions.

   (c) Classify each constant solution as stable, unstable or semistable.

   (d) Suppose $y(t)$ were a population in thousands at time t in years. Furthermore suppose at some instant in time $y = 10$ and then due to some biological consideration the population fluctuated slightly. Explain what would happen.

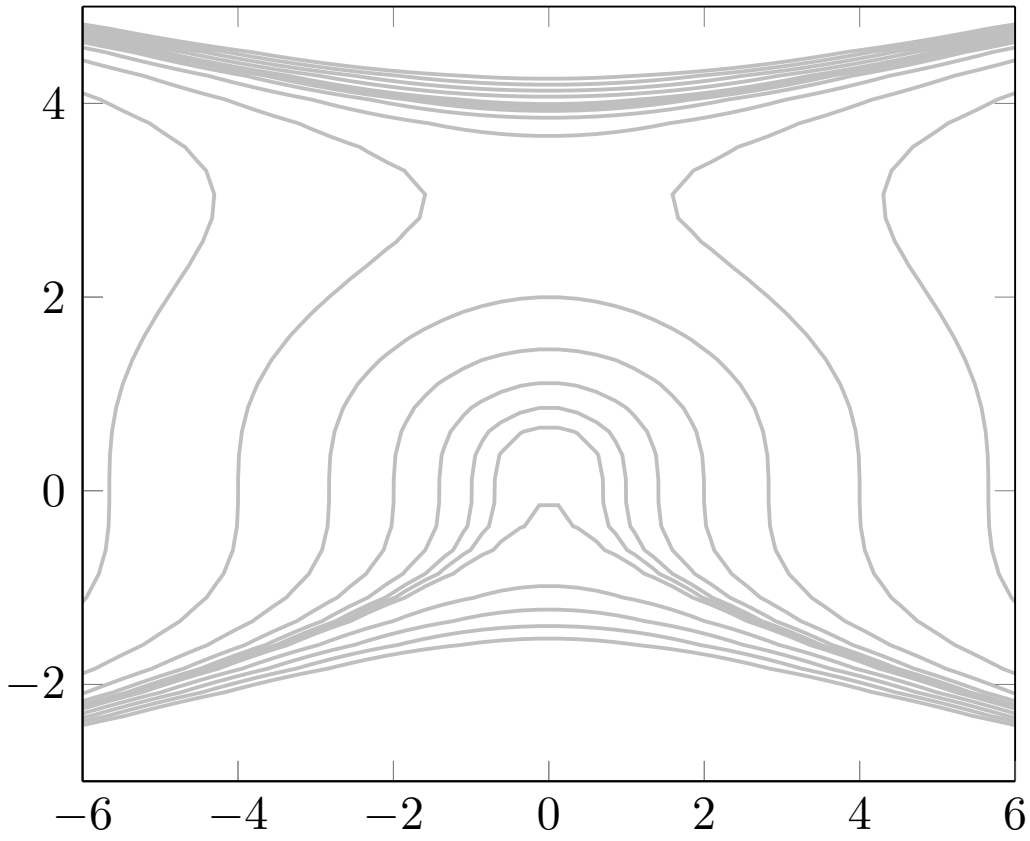
2. The differential equation

$$y' = \frac{t}{y^2(y-3)}$$

has solutions given by

$$y^4 - 4y^3 - 4C - 2t^2 = 0$$

for various C. Plotting a bunch of C values yields:



For (a)-(d) on this graph trace the solutions to the IVP associated with the following initial conditions and label which is which:

(a) $y(0) = 1$.

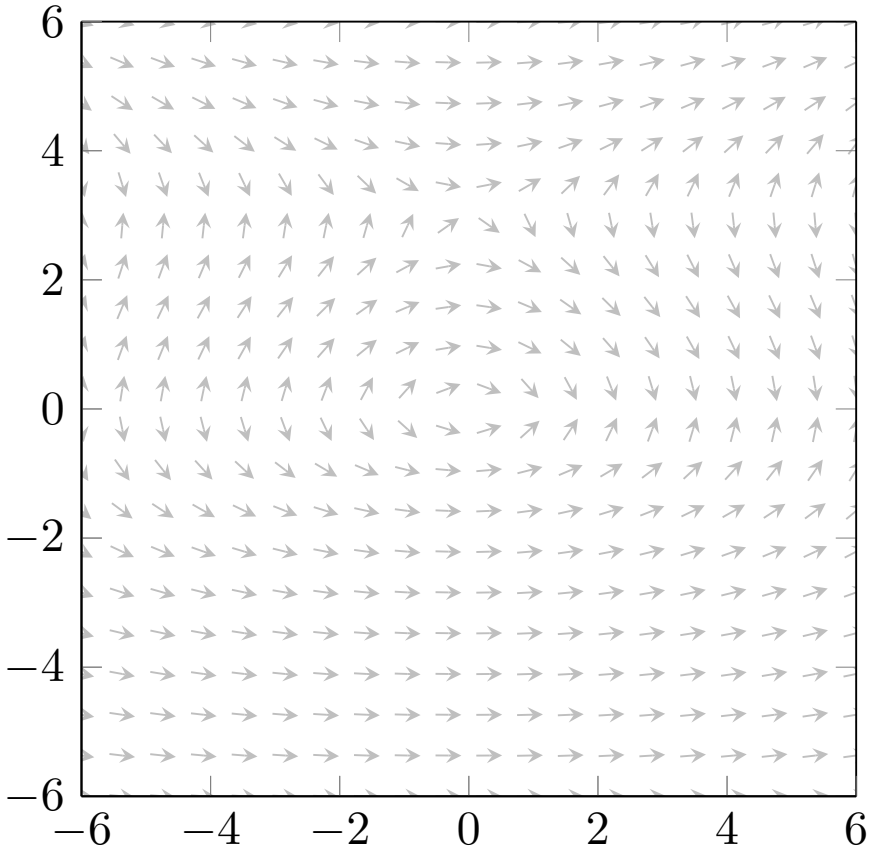
(b) $y(-4) = 1$. What does the interval of existence appear to be?

(c) $y(-4) = -1.5$. What do the coordinates of the relative maximum appear to be?

(d) $y(-2) = 4$

3. The following graph shows the direction field associated to the differential equation

$$y' = \frac{t}{y(y-3)}$$



On this graph trace the solutions to the IVP associated with the following initial conditions and label which is which:

(a) $y(0) = 2$. What does the interval of existence appear to be?

(b) $y(-4) = 4$. What does the interval of existence appear to be?

(c) $y(2) = -1$ What do the coordinates of the relative maximum appear to be?