MATH 246 Homework 2.1 & 2.3
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Directions:

- Work should be done neatly and on separate paper.

- Enough work must be shown so that the steps you are taking is clear.

1. Put the following linear differential equations in normal form and determine the interval of existence (and uniqueness) for the associated initial value problem.

    (a) $y'' + \frac{1}{t-10}y' + \frac{1}{t}y = 0$ with $y(3) = 0$ and $y'(3) = 7$.

    (b) $t\sqrt{5-t}\,y'' + y' + t\sqrt{5-t}\,y = 0$ with $y(2) = 1$ and $y'(2) = -1$.

    (c) $t\sqrt{5-t}\,y'' + y' + t\sqrt{5-t}\,y = 0$ with $y(-2) = 1$ and $y'(-2) = -1$.

2. For each of the following linear systems, if the system is homogeneous determine if it could have nontrivial solutions and if the system is nonhomogeneous determine if it has a single solution or not. Use determinants only.

    (a) The system:

    $$2x + 4y = 0$$
    $$3x + 2y = 0$$

    (b) The system:

    $$10x + 5y + 7z = 0$$
    $$6x + y + 9z = 0$$
    $$2x + 2y - z = 0$$

    (c) The system:

    $$x_1 + 2x_2 = 0$$
    $$x_2 + 2x_3 = 0$$
    $$x_3 + 2x_1 = 0$$

    (d) The system:

    $$4x + 10y = 6$$
    $$2x - 5y = 3$$

    (e) The system:

    $$x + y = 0$$
    $$y + z = 0$$
    $$x + y + z = 1$$

    (f) The system:

    $$2x_1 - 3x_2 + 5x_3 = 8$$
    $$3x_1 - 3x_2 + 9x_3 = 12$$
    $$x_1 - 2x_2 + 2x_3 = 6$$

3. Consider the second order differential equation $y'' + \left(\frac{1}{t-3}\right)y' - t^2 y = 0$.

    (a) Suppose $f(t)$ and $g(t)$ are functions defined on $(-\infty, 3) \cup (3, \infty)$ which satisfy this DE for all those values of t and suppose $f(0) = 3$, $f'(0) = 1$, $f(2) = -1$, $f'(2) = 7$. $g(0) = 3$ and $g'(0) = 1$. What can you conclude about $g(2)$ and $g'(2)$? Explain.

    (b) Suppose all the 2s in the above problem were replaced by 12s. Why can you not draw a similar conclusion? Explain.