Representations of p-adic groups

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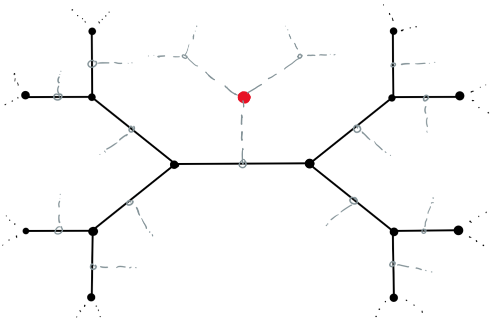
University of Cambridge, Duke University and IAS

November 2020



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- p-adic automorphic forms, p-adic Langlands program
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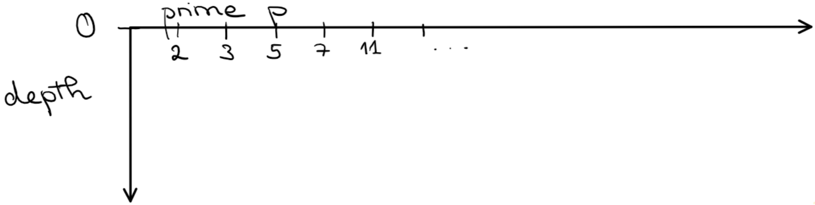

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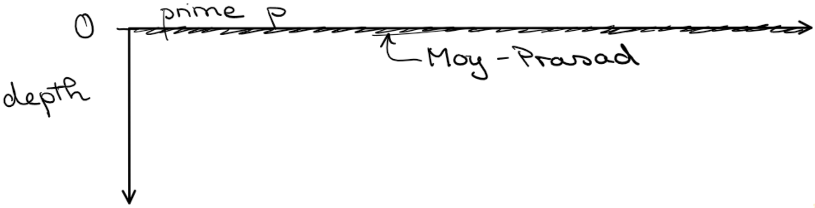

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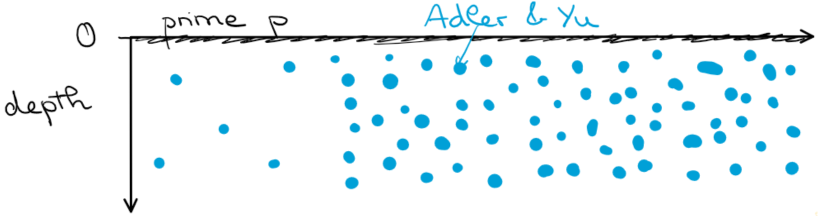

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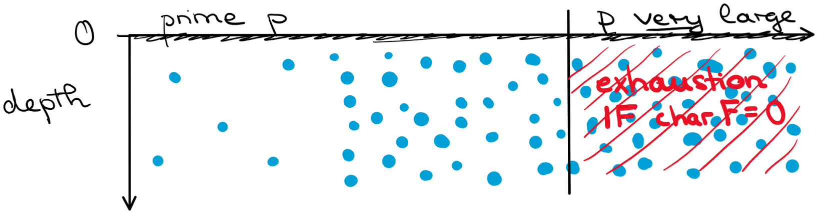

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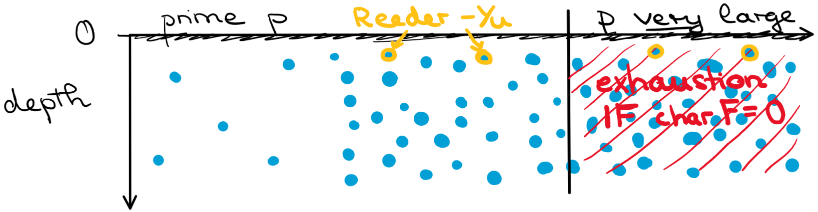

Epipelagic representations

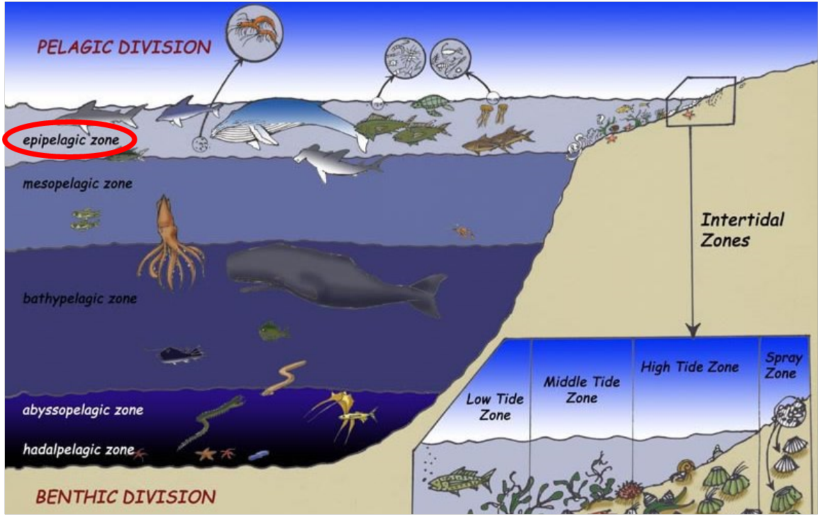

Figure: The epipelagic zone of the ocean;
source: Sheri Amsel. Glossary (what words mean) with pictures!. 2005-2015. April 2, 2015, `http://www.exploringnature.org/db/detail.php?dbID=13&detID=406`

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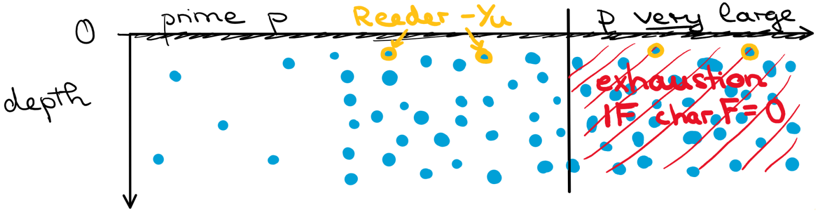

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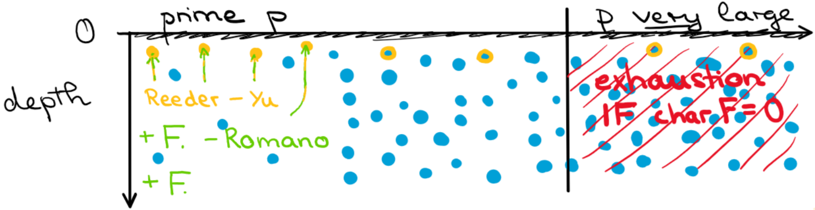

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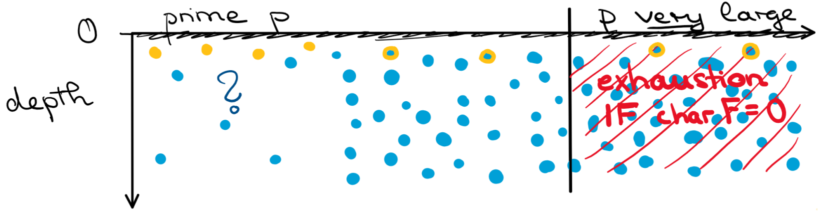

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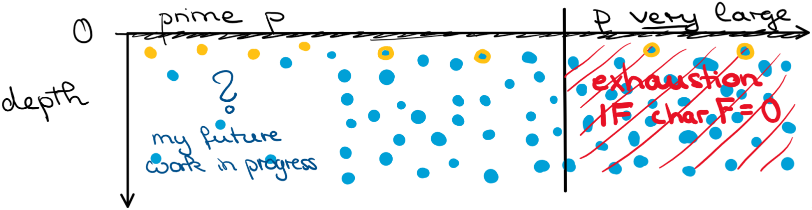

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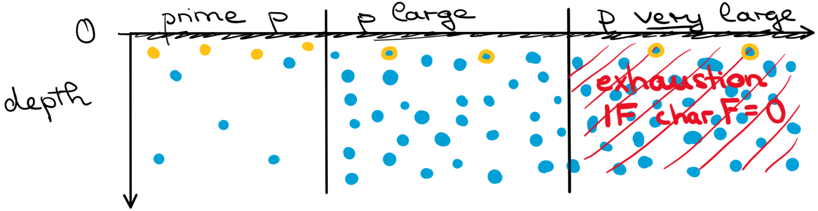

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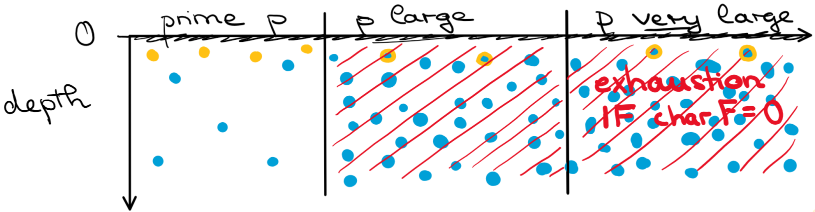

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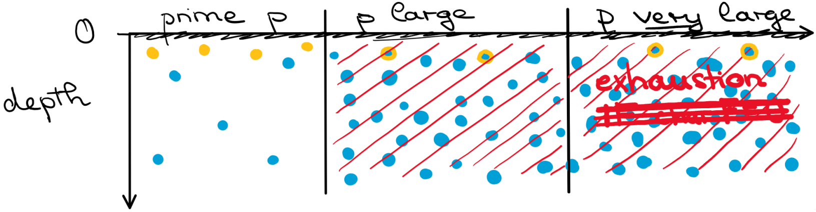

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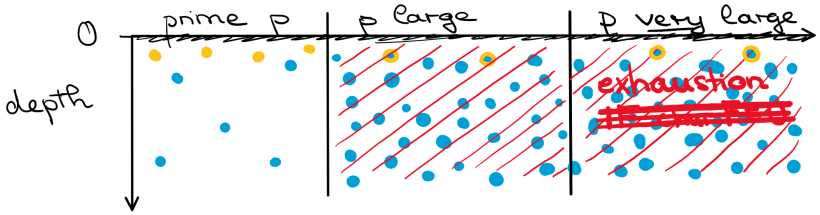
Suppose G splits over a tame extension of F and $p \nmid |W|$, then Yu's construction yields all supercuspidal representations.



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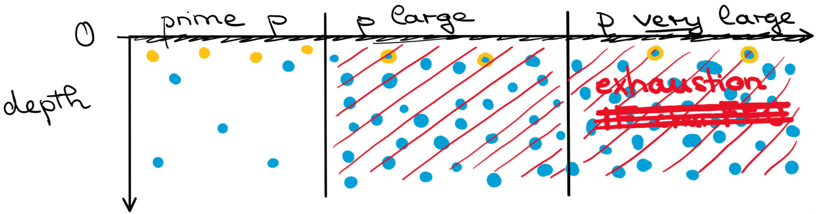
$\lvert W \rvert$	$(n+1)!$	$2^n \cdot n!$	$2^{n-1} \cdot n!$	$2^7 \cdot 3^4 \cdot 5$

type	E_7	E_8	F_4	G_2
$\lvert W \rvert$	$2^{10} \cdot 3^4 \cdot 5 \cdot 7$	$2^{14} \cdot 3^5 \cdot 5^2 \cdot 7$	$2^7 \cdot 3^2$	$2^2 \cdot 3$



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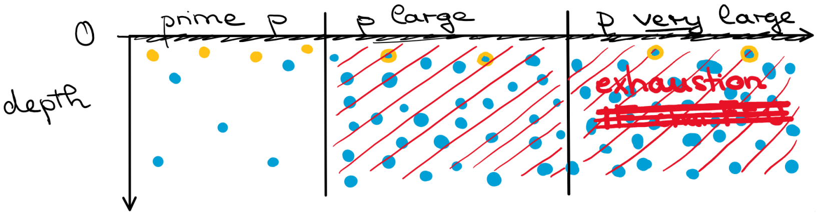

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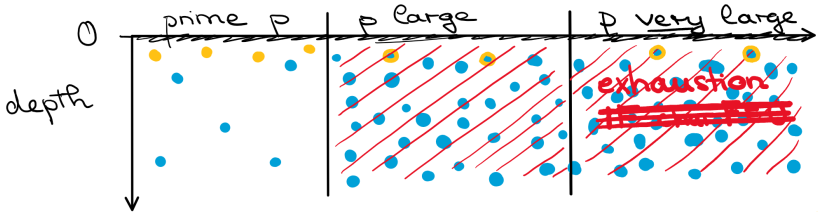

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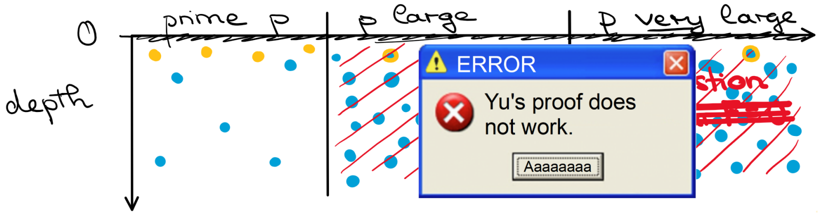

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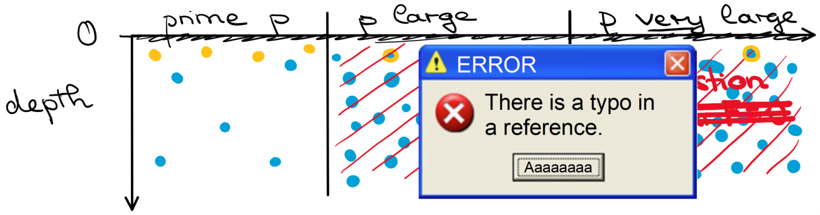

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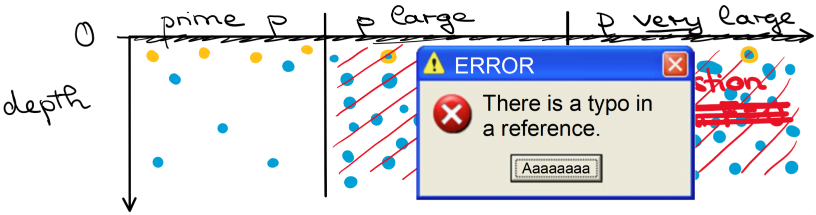

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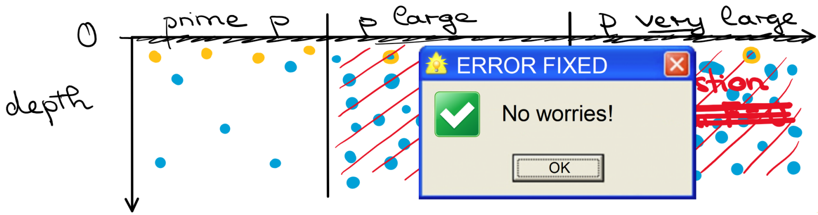

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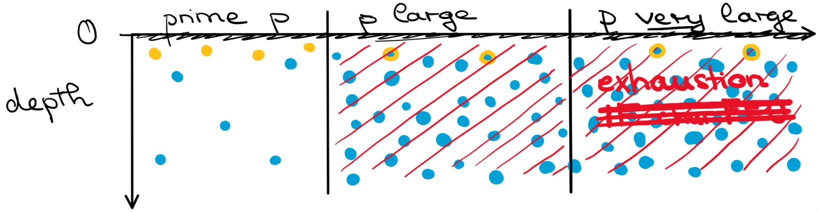

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$$G = \mathrm{SL}_2(F),\ K = \begin{pmatrix} 1+\mathfrak{p} & \mathfrak{p} \\ \mathcal{O} & 1+\mathfrak{p} \end{pmatrix} \times \{\pm 1\}$$

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Supercuspidal representation:

$$\text{c-ind}_K^G \rho_K = \left\{ f : G \to \mathbb{C} \;\middle|\; f(kg) = \rho_K(k) f(g)\ \forall g \in G, k \in K \right\}$$

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Yu's construction and my exhaustion result

$$\begin{aligned} G &= \mathrm{SL}_2(F), \\ x \in \mathcal{B}(G), r &= 0.5, \\ \text{character } &\rho_K \end{aligned}$$

(Reeder–)Yu

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e.g. twist of

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(Reeder–)Yu →

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K, ρ_K such that $\pi := \text{c-ind}_K^G \rho_K$ is supercuspidal

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There exists a counterexample to the key ingredient of Yu's proof (Yu's Prop 14.1 and Thm 14.2, which were based on a misprint).

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$$\epsilon_\sharp^{G/M}(\gamma) = \prod_{\substack{\alpha \in R(T,G/M)_{\mathrm{asym}}/(\Gamma \times \{\pm 1\}) \\ s \in \mathrm{ord}_x(\alpha)}} \mathrm{sgn}_{k_\alpha}(\alpha(\gamma)) \cdot \prod_{\substack{\alpha \in R(T,G/M)_{\mathrm{sym,unram}}/\Gamma \\ s \in \mathrm{ord}_x(\alpha)}} \mathrm{sgn}_{k_\alpha^1}(\alpha(\gamma))$$

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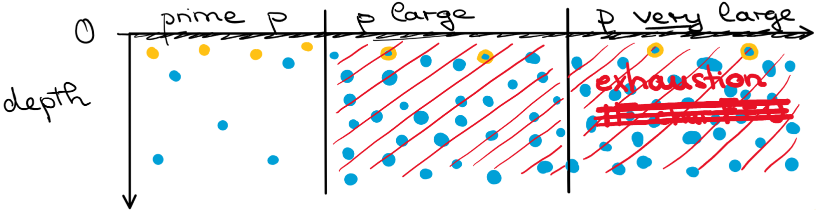

The end of the talk, but only the beginning of the story ...