MATH 634
HARMONIC ANALYSIS

SYLLABUS
Fall 2017

Instructor: John J. Benedetto
Time: Tuesday/Thursday, 12:30 – 1:45
Place: MATH 1311
Prerequisites: Real analysis or permission of instructor.
Office hours: To be settled during the first week of class.
Mandatory departmental in-class test*. Tuesday, October 17,2017.
Grading: Based on homework, midterm, project, and attendance.
Text: *Harmonic Analysis and Applications* (by the instructor) and course notes supplied by the instructor.

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COURSE MATERIAL

1. Fourier analysis on Euclidean spaces including Fourier series
2. Distribution theory and applications
3. Fourier analysis on locally compact abelian groups
4. The uncertainty principle
5. Modern applications of Fourier analysis
6. Discrete Fourier series (DFT) and the Fast Fourier Transform (FFT)
5. Sampling theory and the relations between Fourier transforms, Fourier series, and DFTs
6. Background for time-frequency analysis, wavelet theory on Euclidean spaces and local fields, image processing, dimension reduction, compressive sensing, Wiener's Generalized Harmonic Analysis, and frames

COURSE THEMES
SPECIAL TOPICS FOR PROJECTS

1. The fundamental relation between Fourier analysis and number theory in topics such as the FFT, spectral synthesis, the p-adics, uniform distribution, Kronecker’s theorem, the HRT conjecture, and the Riemann zeta function.
2. Carleson's theorem for Fourier series and recent related research.
3. Algebraic and geometric fundamentals of harmonic analysis, e.g., factorization and automorphisms of group algebras and the characterization of idempotent measures.
4. Beurling algebras, weighted norm inequalities, spectral analysis.
5. Statements and discussions of specific open problems and general unresolved issues: the uncertainty principle, MRI and non-uniform sampling, the Fuglede conjecture and the results of Tao, deterministic compressive sensing and the results of Bourgain, ambiguity functions and Wigner distributions, waveform design and the construction of sequences in terms of Weil’s solution of the Riemann hypothesis for finite fields, the characterization of the space of absolutely convergent Fourier transforms.

6. Since Math 634 is a fundamental course in harmonic analysis, I want to stress that the tentacles of harmonic analysis reach intrinsically into many of the most important applications today.

With regard to the Norbert Wiener Center faculty, which is a subset of the Department of Mathematics, these applications and the NWC's expertise include some of the following cutting=edge topics:

a. Machine and deep learning;
b. Dimension reduction with its relations to geometrical properties of lower dimensional manifolds;
c. Single pixel cameras;
d. Super-resolution in image processing and medical applications;
e. Phaseless signal reconstruction of which Hauptmann, a UMD MATH PhD, received the Nobel prize!;
f. Frame theory, which is the most natural and sophisticated mathematical model for addressing omnipresent noise reduction problems;
g. Data science.

Without exaggerating, the list goes on and on.

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- This will be an open book 50 minute test based on your graded homework. Besides the text you may take the following into the classroom for the test: all notes, homework, etc., that you have from the course.