Ten Books on Wavelets

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Wavelets: a Primer. *By C. Blatter*. A K Peters Ltd, Natick, MA, 1998. $32.00. x+202pp., hardcover. ISBN 1-56881-095-4.
Wavelets: a Mathematical Tool for Signal Analysis. *By C.K. Chui*. SIAM, Philadelphia, PA, 1997. xi+210pp., softcover. ISBN 0-89871-384-6.
Wavelet Toolware: Software for Wavelet Training. *By C.K. Chui, A.K. Chan, and C.S. Liu*. Academic Press, N.Y., 1998. ISBN 0-12-174595-3.
Fourier Analysis and Applications: Filtering, Numerical Computation, Wavelets. *By C. Gasquet and P. Witomski*. Springer Verlag, N.Y., 1998. Translated by R. Ryan. xviii+442pp., hardcover. ISBN 0-387-98485-2.
Wavelets, Approximation, and Statistical Applications. *By W. Härdle, G. Kerkyacharian, D. Picard, and A. Tsybakov*. Springer Verlag, N.Y., 1998. xviii+265pp., softcover. ISBN 0-387-98453-4.
Real Analysis - with an Introduction to Wavelet Theory. *By S. Igari*. AMS Translations of Mathematical Monographs, volume 177, AMS, Providence, RI, 1998. $89.00. Translated by S. Igari. xiii+256pp., hardcover. ISBN 0-8218-0864-8.
Wavelets and their Applications: Case Studies. *By M. Kobayashi, editor*. SIAM, Philadelphia, PA, 1998. xvi+142pp., softcover. ISBN 0-89871-416-8.
A Wavelet Tour of Signal Processing. *S. Mallat*. Academic Press, San Diego, CA, 1998. $59.95. xxi+577pp., hardcover. ISBN 0-12-466605-1.
Wavelet Analysis: the Scalable Structure of Information. *By H.L. Resnikoff and R.O. Wells, Jr.*. Springer Verlag, N.Y., 1998. $59.95. xii+435pp., hardcover. ISBN 0-387-98383-X.
Image Processing and Data Analysis: the Multiscale Approach. *By J.-L. Starck, F. Murtagh, and A. Bijaoui*. Cambridge University Press, 1998. $34.95. x+287pp., softcover. ISBN 0-521-59914-8.
A Mathematical Introduction to Wavelets. *By P. Wojtaszczyk*. London Mathematical Society, Student Texts 37, Cambridge University Press, 1997. $24.95. xii+261pp., softcover. ISBN 0-521-57894-9.

The Editor of Book Reviews for SIAM asked me to review these books, and to provide some background material. I accepted his request with the proviso that I could use the title above. (One of the items is a software package, and so there are "only" ten books.) The title refers to Daubechies' book *Ten Lectures on Wavelets* [Dau92] and to those exhilarating nascent days of wavelet theory, when Meyer's *Ondelettes et Opérateurs* [Mey90] as well as [Dau92] established the field. The books under review here are really the tip of the iceberg, and any number of journals could have similarly titled reviews without any overlap of books.

What is a *wavelet*? In the setting of $L^2(\mathbb{R})$, the space of square integrable functions on the real line $\mathbb{R}$, a *wavelet* ψ is an element of $L^2(\mathbb{R})$ with the property that the *wavelet system* $\{\psi_{m,n} : m, n \in \mathbb{Z}\}$ is an orthonormal basis (ONB) for $L^2(\mathbb{R})$, where $\psi_{m,n}$ is defined as

$$\psi_{m,n}(t) = 2^{m/2}\psi(2^m t - n). \tag{1}$$

Thus, each $f \in L^2(\mathbb{R})$ has the wavelet expansion

$$f = \sum_{m,n\in\mathbb{Z}} \langle f, \psi_{m,n}\rangle \psi_{m,n} \quad \text{in} \quad L^2 - \text{norm},$$

where the inner products $< f, \psi_{m,n} >$ are the *wavelet coeffiecients*.

What is *wavelet theory*? This is a much bigger question, that was first answered in [Mey90] and [Dau92]. It continues to be answered in diverse ways, as existing methods interact with other branches of mathematics, as the first rush of wavelet results has had a chance to regroup and evolve and mature, and as new applications have tested and created wavelet-based algorithms. Wavelet theory has developed into an imposing mathematical edifice with vitality and depth, as well as with some emerging limitations and baroque tendencies. The scope of its applicability exhibits a similar effectiveness and limitation. Further, it is a relief to assert that all that glitters is not a wavelet! Not that anyone ever said that wavelets were a panacea, but there was definitely a period of over-prescription of them.

For perspective in dealing with the books under review, especially considering their diverse subject matter, it is reasonable to sketch some of the background to [Mey90] and [Dau92], as well as to highlight some important works from the mid-1990s.

The first wavelet ONB was constructed by Haar [Haa10] in his thesis. The *Haar wavelet* h for the setting of $L^2(\mathbb{R})$ is defined as

$$h(t) = \begin{cases} 1 & \text{if } t \in [0, 1/2), \\ -1 & \text{if } t \in [1/2, 1), \\ 0 & \text{otherwise.} \end{cases}$$

Haar's work was followed by Walsh's construction [Wal23] in 1923 of ONBs in terms of so-called *Walsh functions*. Actually, about 1900 and without any interest in ONBs, engineers designed transposition schemes in open-wired lines based on Walsh functions, and they used these schemes to minimize channel "crosstalk". The sequence of Walsh functions on $\mathbb{R}$ is the prototype of *wavelet packets*, just as the Haar wavelet system $\{h_{m,n}\}$ is the prototype of wavelet ONBs on $\mathbb{R}$. The theory of wavelet packets is due to Coifman, Meyer, and Wickerhauser, e.g., [CMQW90], [CMW92], [Wic94].

It would be nice to give a sequential litany of waveleteers, beginning with Haar and Walsh, and taking us to Meyer and Daubechies. Unfortunately, it didn't happen linearly. It is true that the construction of wavelet ONBs is part of a program to construct unconditional bases for many of the important function spaces in analysis; and hence there is a certain lineage after Walsh from Franklin (1928), to Pelcynski and Ciesielski, to Carleson, and to Strömberg [Str83]. However, there were many other paths to the establishment of wavelet theory, not all of which were fully appreciated by the late 1980s. Briefly, and before the appearance of [Mey90] and [Dau92], there were wavelet-oriented traditions and/or developments in spline and approximation theory, in speech and image processing, and in atomic decompositions and the Calderón reproducing formula.

Further, because of the role of Fourier analysis in wavelet theory, it is often reasonable to include Fourier-based decompositions in the same framework as wavelet expansions. As such, other notions leading to this broader view of wavelet theory include the ambiguity function and the Wigner transform, as well as the short time Fourier transform (STFT) and the coherent state decompositions from quantum mechanics which lead to the Weyl-Heisenberg or Gabor theory of signal decomposition. The origins of this latter theory are found in von Neumann's monograph [vN55], which was written in the late 1920s, and some of the basic ideas are due to Dennis Gabor [Gab46] in 1946, see [FS98] for recent developments. Gabor received the Nobel prize in

1971 for his conception of holography, which he formulated during this same period in the 1940s. Because some aspects of holography can be modelled in terms of the Heisenberg group, it is not unreasonable to speculate on an intellectual relationship between holography and Gabor's theory of signal decomposition. In fact, the geometric invariants of Fourier analysis can be split into underlying actions determined by the affine group, which determines wavelet theory, and the Heisenberg group, which determines Gabor theory. These groups can be integrated into the study of the metaplectic group, see [HL95] for a mathematical analysis and [BJ92], [MH92] for some engineering perspective.

In any case, the following remarks illustrate some of the intellectual intricacy leading to wavelet theory.

Carleson's construction of an unconditional basis for the Hardy space H^1 in 1980 led Strömberg to construct his spline wavelet ONB in 1981 [Str83]. Using cardinal B-splines, Battle and Lemarié independently constructed wavelet ONBs in the late 1980s *in the context of* wavelet theory. The connection between splines and wavelets continues to the present day, and some of the present books under review trace recent developments, e.g., the books by Chui, Mallat, and Wojtaszczyk. Further, there are natural relationships between other aspects of approximation theory and wavelet theory, e.g., [DJP92].

In harmonic analysis, the atomic decomposition theory of Coifman [Coi74] and Coifman and Weiss [CW77] for the Hardy spaces H^p, $0 < p \leq 1$, also had an influence on the development of wavelet theory at the level of expansions in terms of "atoms" (harmonics) having vanishing moments. It is natural, but not necessary, that the harmonics $\psi_{m,n}$ of a wavelet expansion have vanishing moments. The Calderón reproducing formula [Cal64] is another influence on wavelet theory from harmonic analysis, and it is now synonomous with the so-called *continuous wavelet transform* (CWT). Calderón's formula is

$$f(t) = \int_0^\infty (\psi_{1/u} * \psi_{1/u} * f)(t) \frac{du}{u}, \tag{2}$$

where the dilation ψ_x is defined as $\psi_x(t) = x\psi(tx)$ for $x > 0$. This formula is a continuous ("overcomplete") wavelet decomposition of $f \in L^2(\mathbb{R})$. We have used the term "wavelet" in the sense of describing f as an integral ("sum") whose harmonics are dilates and translates of a fixed function ψ, cf., (1). In fact, equation (2) is

$$f(t) = \int_0^\infty \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \psi_{1/u}(v-w) f(w) dw \right) \psi_{1/u}(v-t) \frac{dvdu}{u},$$

and so the "sum" we have alluded to is the double integral

$$\int_0^\infty \int_{\mathbb{R}} \cdots \frac{dvdu}{u} \tag{3}$$

taken over the temporal or spatial variable v and the dilation u. In this case the "wavelet coefficients" of f are

$$\int_{\mathbb{R}} \psi_{1/u}(v-w) f(w) dw, \quad v \in \mathbb{R}, \quad u > 0.$$

The formal verification of Calderón's formula is elementary, as long as ψ satisfies certain properties. In fact, the Fourier transform of the right side of equation (2) is

$$\widehat{f}(\gamma) \int_0^\infty \widehat{\psi}(u\gamma)^2 \frac{du}{u},$$

where $\widehat{f}$ designates the Fourier transform of f; and there are even, real-valued, compactly supported, infinitely differentiable functions ψ for which

$$\forall \gamma \in \mathbb{R} \backslash \{0\}, \quad \int_0^\infty \widehat{\psi}(u\gamma)^2 \frac{du}{u} = 1,$$

e.g., [FJW91].

There are overcomplete wavelet decompositions where the integrals in (3) are replaced by summations over the integers. In this case we obtain a *wavelet frame* decomposition of $f \in L^2(\mathbb{R})$. A wavelet frame is an important example from the general theory of frames, originally formulated by Duffin and Schaeffer [DS52] to study non-harmonic Fourier series. This intellectual journey began with a result by G.D. Birkhoff on Sturm-Liouville systems, and was followed by fundamental work of Paley-Wiener (1934), Levinson (1940), Beurling-Malliavin (1960s), and Henry Landau (1967). There is presently a resurgence of these ideas in the context of irregular sampling. Work of Daubechies, Grossmann, and Meyer (1986) led to the interleaving of frames and wavelets. There is a presentation of frames in this context in [Dau92], cf., [You80]. The 1990s have seen a great deal of activity, both theoretical and genuinely applicable, in the theory of frames.

We also mentioned speech and image processing as areas whose methods and points of view played a role in the development of wavelet theory. In fact, it is not an exaggeration to say that some of these techniques and concepts, developed in the late 1970s and early 1980s, are now fundamentals of wavelet theory. Briefly, Burt and Adelson [BA83] formulated a Laplace pyramidal scheme to remove redundant image correlation of neighboring pixels as an image compression technique, and there are related ideas in general image processing dealing with multiscale analysis, e.g., [Mar83]; all of which is intimately related to multiresolution analysis (MRA). Similarly, quadrature mirror filters (QMFs) were introduced in the late 1970s to deal with speech coding problems. Some of the basic results are found in [CEG76], [CWF76], [EG77], and [SB86]. These filters are a major component in the multiresolution analysis algorithm (and theorem) to generate wavelet ONBs. In fact, every MRA defines a QMF, whereas there are QMFs which do not arise from MRAs. Therefore, in a formal sense, the notion of a QMF is more general than an MRA. The reality is more complicated. For example, there are single functions $\psi \in L^2(\mathbb{R}^d)$ for which $\{\psi_{m,n}\}$, properly modified to account for dimensionality, is an ONB for $L^2(\mathbb{R}^d)$, but where $\{\psi_{m,n}\}$ does not arise from an MRA. This fact is particularly surprising and striking in the case $d > 1$, e.g., [DL98].

I taught my first wavelet theory course in January of 1987. I was armed with several exciting manuscripts by Meyer, viz., [Mey85], [Mey86a], and [Mey86b], as well as [GM84], [GMP85], [LM86], and [DGM86] by Daubechies, Grossmann, Lemarié, Meyer, Morlet, and Paul. Later that semester, I attended Meyer's Zygmund Lectures on *Wavelets with compact supports*. During those days, I found out about Daubechies' wonderful, just-proved theorem in which she constructed smooth compactly supported wavelets. Later that year, I discovered frames, and eventually realized that Frazier and Jawerth [FJ85] had accomplished wavelet frame decompositions very early-on and in the context of the Littlewood-Paley theory. Frazier and Jawerth didn't have orthonormality or necessarily unique representation, but we know that such "redundant" decompositions as theirs are desirable and applicable in many situations. Similarly, during the same period, Feichtinger and Gröchenig [FG89] produced early drafts of their general theory which included some important decompositions that are an integral part of what is now called wavelet theory. My own efforts, to put in perspective those notions from speech and image processing mentioned above, would have been minimal if only I had known of the early work by Vetterli or if the beautiful books by Strang and Nguyen [SN96], Vetterli and Kovačević [VK95], and Vaidyanathan [Vai93] had existed then.

Nowadays, I can't keep up with all of the new wavelet books. There are those that exist (I'm sure), that I've not heard of; there are those I've heard of, that I've not seen; there are those I've seen, that I have not read; and there are those that I've read - or at least read parts of them! In this last category, and besides the other books quoted in this review, I have enjoyed [CR95],[Fra99],[HW96], [Hol95], and [Mey93]. There are also some excellent edited volumes.

Thus far, I've talked about the background to the onset of wavelet theory, and hinted at some recent activity. It would be propitious for the reviews that follow to describe the present landscape of wavelet theory, but it may be a foolhardy task for this reviewer. I refer instead to several wellsprings of information. The Wavelet Digest (http://www.wavelet.org) provides the latest in professional activities. The journals, Applied and Computational Harmonic Analysis (ACHA) and Fourier Analysis and Applications (JFAA), are major reading places for all facets of wavelet and Gabor theory. SIAM publishes many books and articles in the field, and the IEEE Transactions in both Information Theory and Signal Processing are also important sources of results.

I began by asking "what is wavelet theory?". For me, but not for everyone, it is a branch of harmonic analysis and an effective tool for certain problems in signal processing. Branches often beget new branches and good tools often beget new ideas. Wavelet theory itself has certainly begotten new books.

In reviewing the following books, I have asked some of my students to share their experiences with the books as far as readability and relevance and difficulty of exercises with respect to the text. It is a pleasure to acknowledge Amit Banerjee, Matt Fickus, Hui Ji, Edwin Johnson, Alfredo Nava-Tudela, Götz Pfander, Yuan Qi, Anwar Saleh, Sherry Scott, Oliver Treiber, and Shijun Zheng. Several of the books under review are not really devoted to wavelet theory, e.g., Igari's book; and they deserve, and will probably receive, reviews based on their central themes. Several of the genuinely wavelet theory books are substantial and/or innovative, and deserve separate reviews.

Wavelets: a Primer by Blatter is an introduction to wavelet theory for mathematicians, and provides the basic results in a clear manner. It is suitable as a quick reference, and to illustrate some essential ideas. There are no exercises or applications. The examples are generally limited to those associated with new concepts, and are not as numerous as the back cover advertises.

Wavelets: a Mathematical Tool for Signal Analysis by Chui also presents the basic theory in a clear, crisp manner, introducing various spline wavelets in a motivating fashion and integrating signal processing terminology in the theoretical presentaion. The basic theory includes a mandatory discussion on "what are wavelets" and a chapter on time-frequency localization. The author then has chapters on MRA, orthonormal wavelets, biorthogonal wavelets, and algorithms. Unfortunately, there are no exercises.

The last fifth of the book is devoted to the important signal processing applications of detection of singularities and data compression, and to the numerical solution of integral equations. Although the author is an authority on frames, he does not deal with their theory or applications in this book. He does include a list of commercial and public domain wavelet software, but surprisingly fails to mention explicitly the MATLAB Wavelet Toolbox.

The author has been an important contributor to wavelet theory for many years, and the book is both readable and written with significant understanding.

Wavelet Toolware: Software for Wavelet Training by Chui, Chan, and Liu is a companion software package to the wavelet best-seller *An Introduction to Wavelets* by the first-named author. A 73 page booklet is provided with the CD-ROM. The first and second sections of the booklet contain basic wavelet theory and algorithms, respectively. The third section is a user's manual. It contains the technical requirements for installation, and instructions for implementing the wavelet software; and it displays various pictures of the graphical user interface. Capabilities to add user-defined processing functions are a good feature. Also, the format of the data input is given, and this allows for user-defined data sets to be generated from other sources.

The program itself is intuitive in its interface and is self-explanatory. When computing the CWT, we found that a pop-up message warning of the possible delay in the computation has to be acknowledged in order for the actual computation to get started. If one fails to do so, the computation will indeed seem quite long since it doesn't start at all. The technical requirements to run the program are not well detailed on the packaging box that contains both the book and the software. Finally, in its present form, the software is a subset of the capabilities built into the MATLAB Wavelet

Toolbox.

Fourier Analysis and Applications: Filtering, Numerical Computation, Wavelets by Gasquet and Witomski is a mathematics textbook in Fourier analysis oriented towards issues in signal processing. The translation by Robert Ryan from the original French edition is excellent. There are chapters on integration and functional analysis, as well as a treatment of Fourier series, Fourier transforms, DFTs and FFTs, and distribution theory. The authors also include chapters on filters and sampling, and the material is well-integrated and accompanied by a good selection of exercises. The material is essential Fourier analysis, and there are several other very good recent books in the area. Wavelet theory only occupies the last 35 pages in a book of 442 pages. This is not necessarily bad, just a fact. The authors provide several levels of presentation for classroom use; and the book could be used at senior level through to the second year of graduate school. The material is well-organized into 42 lessons.

Wavelets, Approximation, and Statistical Applications by Härdle, Kerkyacharian, Picard, and Tsybakov grew out of a French-German seminar bringing together theoretical and applied statisticians from Berlin and Paris. There are 12 chapters, the first 9 of which deal with wavelet theory. Chapters 8 and 9 discuss wavelets and approximation theorems in Sobolev and Besov spaces. The exposition of wavelet theory accounts for about half of the book. The whole book is well-written and readable, and there are interesting examples from the area of finance, e.g., [Fam76]. There are no exercises.

Chapters 10-12 exposit the use of wavelets in nonparametric statistical estimation problems. Nonlinear thresholded wavelet estimates play a role in such problems, and the authors give a thorough treatment of thresholding in Chapter 11. The work of Donoho and of Donoho and Johnstone is duly advertised, but some other related wavelet statistics work is omitted, e.g., the results of Houdré and Cambanis [Hou94], [CH95]. The final chapter is devoted to computational algorithms and statistical software implementations. As a non-statistician, the material on Stein's method of unbiased risk estimation and the inequalities in Appendix C were illuminating.

Real Analysis - with an Introduction to Wavelet Theory by Igari is mostly a book on real analysis with few exercises. In fact, the first 6 chapters take the reader through the Lebesgue theory at a sophisticated level in a conventional way. The author then treats distribution theory in 15 pages, Fourier analysis in 44 pages, and wavelet theory in 28 pages. There are no exercises on wavelet theory. The presentation is clear and rigorous; it is "polished", to quote from the author's Preface to the English Edition. I don't think it is unimportant to write books where wavelet theory takes the form of a peroration or even as a small but important part of one's more general mathematical or scientific point of view. Igari's book presents wavelet theory in both of these ways.

Wavelets and their Applications: Case Studies, edited by Kobayashi, is part of a collection of works which were featured in the *Wavelets in Japan* series in SIAM News. There are 5 case studies, and Kobayashi has contributed to two of them. This is not a textbook and so there are no exercises. The writing is clear, and the lists of references introduced me to many Japanese contributors to wavelet theory. For an electrical engineer, trained for digital signal processing (DSP) applications using multirate systems, this book is a breath of fresh air. The applications deal with partial differential equations, seismology, topics from mechanical and nuclear engineering, and, comfortingly, for the aforementioned electrical engineer, a case study on noise reduction. Of course, there have been non-DSP applications of wavelet theory since its modern inception by Morlet for ELF Aquitaine, but DSP has been a dominant venue for wavelet R&D. In any case, the full power of the modern power-couple of wavelets and DSP is not reflected in Kobayashi's experiment-oriented presentation.

The first case study begins with the following problem. "Given Japanese coastline point data, display a view of the coastline at various resolution levels as quickly and accurately as possible on a workstation or PC." The authors convincingly argue against using fractal curves or splines, and justify their use of wavelets. They then implement Mallat's MRA algorithm [Mal89a], and analyze approximations for curves and coastlines using different wavelets. There are no theorems to assert optimal results.

The noise reduction technique introduced in the second case study is a simple adaptation of Mallat's algorithm, and it works well for the author's experiments dealing with energy absorption properties of silicone and with studying properties of dry friction. The author even documents a failure of the method. The third case study introduces a wavelet-based preconditioning method for solving large systems of linear equations. It also provides an error analysis for dealing eventually with real fluid dynamics problems. The fourth case study deals with text-to-speech systems, and uses splines to analyze pitch.

The last case study is a wavelet analysis of wind and turbulence velocity data. Numerical simulations using the Navier-Stokes equation were used to produce the velocity data, and the authors' analysis uses various wavelet methods as well as some other notions from harmonic analysis. They describe several wavelet-based numerical methods for solving differential equations in this area, but fail to mention the work of others such as Farge and Wickerhauser.

A Wavelet Tour of Signal Processing by Mallat will join the ranks of [Mey92] and [Dau90] as a central, inclusive, monumental, and seminal treatise on wavelet theory. (Waxing rhapsodic, the qualifier "millenial" also occurred to me.) At the risk of making arbitrary distinctions about categories whose very existence is questioned in certain company, Meyer's treatise is pure mathematics, Daubechies' treatise is applied mathematics, and Mallat's treatise is mathematical signal processing. They are all fundamental books in mathematics. Mallat has been involved with many of the most innovative wavelet ideas, both theoretical and applicable, from the late 1980s with the now classical Mallat-Meyer MRA theory and algorithm, e.g., [Mey86b], [Mal89a], [Mal89b], to the present day. His book reflects his fundamental ideas at the origin of the subject as well as his scientific point of view in terms of digital image processing. It also reflects a catholic point of view by making a serious multidisciplinary exposition.

Mallat has not only written a treatise, but also an excellent graduate text for students in computer science, electrical engineering, and mathematics. How can a book of 577 pages be a textbook? Mallat answers this question in the introduction and at the end of Chapter 1, and the answer is that he has has put all of the material into one or the other of three levels: central ideas, material dedicated to an application, results at the frontier of research. He presents ideas and techniques, theorems and applications. The material is extremely well-organized and very well-written. Chapter 1 is entitled "An introduction to a transient world". This is not morose philosophy about our banal and superficial existence, but a lively, clearly presented preview of the existence and importance of transient signals, and how they are going to be analyzed in terms of wavelet theory and signal processing. There are 11 chapters, 2 appendices on some mathematics topics and software toolboxes, and a bibliography listing 71 books and 316 articles - long bibliographies seem to be a signature of wavelet books (and reviews!). The exercises are wonderful. They range from elementary to difficult, from mathematical and computational to project-oriented. Many of the exercises suggest implementations in WaveLab, which is a freeware library of MATLAB routines developed by Donoho et al. I should mention that the exercises in [VK95] are also excellent.

The chapter titles are major areas in wavelet theory: 2. Fourier kingdom, 3. Discrete revolution, 4. Time meets frequency, 5. Frames, 6. Wavelet zoom, 7. Wavelet bases, 8. Wavelet packet and local cosine bases, 9. An approximation tour, 10. Estimations are approximations, and 11.Transform coding. Each chapter contains the general theory and algorithms one would expect by their titles, but there is also a great deal of information that one might not expect. There are excellent treatments of instantaneous frequency and wavelet ridges, of lifting, of edge detection, of signal compression, and of multifractals. Mallat's own contributions on MRA, matching pursuits, wavelet transform modulus maxima, image processing, etc. are integrated into the general theory, and not in a lopsided way. The role of Besov spaces and frames and time-frequency distributions and many other topics are explained.

Mallat's second edition will include a treatment of functions of bounded variation for image processing, and the minimax theory for denoising and video compression. I look forward to SIAM 's review as a feature attraction!

Wavelet Analysis: the Scalable Structure of Information by Resnikoff and Wells is advertised by the publisher as a text for upper-level undergraduate or beginning graduate students in engineering or mathematics. In this regard, although there are a fair number of examples and detailed calculations, there are no exercises. The authors express their gratitude to Wayne Lawton for his collaboration in Chapter 7, to Michael Tzannes for his contributions to Chapter 14, and to Peter Heller who contributed extensively to Chapters 4, 5, and 6. The book is divided into four parts: I. The scalable structure of information (Chapters 1-3), II. Wavelet theory (Chapters 4-7), III. Wavelet approximation and algorithms (Chapters 8-12), IV. Wavelet applications (Chapters 13-14). These applications deal with the important topics of data compression and channel coding. Granted that the book is not a textbook (I don't think the authors say that it is), it most certainly is a highly professional mathematics book on wavelet theory, and the authors have made contributions to the field for many years.

Notwithstanding the conventional "wavelet title" of parts II, III, and IV, this is a book with a personality and point of view. The stage is set in part I, which is brief (36 pages) and relatively discursive. The authors take up the theme of *mathematical engineering*, a term which I think is due to Kailath, and motivate the MRA approach to wavelet theory by illustrating the multiscale nature of information "in the mathematical and natural worlds".

Wavelet theory (Part II) is presented in terms of the authors' theory of *wavelet matrices*, which was developed with Peter Heller in the early 1990s. The treatment is thorough and up-to-date. Part III gives a comprehensive treatment of Mallat's MRA algorithm, as well as a detailed discussion of wavelet discretization for differentiation. This latter topic is a valuable contribution, and is used in the authors' exposition of wavelet-Galerkin solutions of partial differential equations. The presentation of applications in Part IV is clear, introductory, and excellent. This is another book which deserves an in-depth review.

Image Processing and Data Analysis: the Multiscale Approach by Starck, Murtagh, and Bijaoui deals with multiscale methods for a host of applications; and, although Mallat's algorithm is established early on, wavelet techniques are used, quite properly, as part of a broader signal processing approach. There are 9 chapters, and Chapters 5 and 6 are due to Jean-Pierre Djamdji. This is not a textbook, and the overall presentation is clear, although I was confused by some specific explanations, e.g., on disparity analysis in remote sensing.

Broadly speaking, the authors study the use of multiscale methods for image enhancement techniques, such as noise removal and deconvolution, and for image analysis applications, such as image registration, stereo vision, and object detection. Satellite and astronomic images constitute much of the analyzed data. Besides Mallat's algorithm (1989), the authors also deal with Feauveau's algorithm (1990), the *à trous* algorithm of Holschneider et al. (1989), and Burt and Adelson's Laplace pyramidal scheme (1983). These multiscale methods are integrated with other signal processing methods in a very nice way on several occasions. For example, in Chapter 2 the authors show the efficacy of Wiener filtering in wavelet space for purposes of noise removal; and in Chapter 3 the Maximum Entropy Method is used for image restoration in conjunction with MRA.

Although I am an amateur about the real-world data of the last three chapters, I wish that the authors had also made their analysis with more recent methods. (Of course, perhaps their present results would still 'win' for the specific data at hand.) For example, Chapter 7 deals with image compression, and, in the last decade, there has been significant progress in the development of wavelet-based image and video compression methods, buiding on the energy compaction property of the wavelet transform. Also, in several instances throughout the book, it is not clear that the multiscale methods provide the optimal route; and some compelling analysis would have been desirable. Having said all of that, I think the book is original, and is an example to build on for forthcoming excursions in mathematical engineering.

A Mathematical Introduction to Wavelets by Wojtaszczyk is a graduate level mathematics textbook in wavelet theory. The exercises are just right, as far as their number, difficulty, and appropriateness to the text. There are 9 chapters and 5 short appendices on topics in analysis and notation for spaces. The author develops wavelet theory in the standard way ("standard" is not bad). The writing is clear, and the proofs are complete.

In the first 4 chapters, the author constructs the Haar and Strömberg wavelets, develops the Meyer-Mallat MRA theory, defines Meyer, spline, and unimodular wavelets, and gives Daubechies' construction of smooth compactly supported wavelets. The treatment in Chapter 5 of multidimensional MRAs and related notions is superb. The author develops the theory for expansive matrices, which includes the multidimensional dyadic case, and establishes the required number of wavelets to generate an ONB in this case. He also proves the basic results from the Gröchenig-Madych theory [GM92], beginning with generating MRAs whose scaling functions are characteristic functions, cf., [CD93].

The remainder of the book is a serious treatment of wavelet expansions and their relations to some standard spaces in analysis, including L^p, H^1, BMO, and Besov spaces, this last of which brings us back to [DJP92] and [FJ85].

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