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8. Brief biography:
John Benedetto is an harmonic analyst. He has published over 100 papers, including 9 monographs, in wavelet and frame theory, in spectral analysis, estimation, and synthesis, and on the uncertainty principle, speech compression, and noise reduction. He has had 20 Ph.D. students, is Editor-in-Chief of the Journal of Fourier Analysis and Applications, and is the Series Editor of Birkhauser's Applied and Numerical Analysis series.

Wavelet periodicity detection algorithms

Abstract

We shall address the problem of detecting periodic signals embedded in generally non-periodic time series. Examples abound, including the epileptic seizure prediction problem for EEG and ECoG data, the problem of characterizing specified water waves in turbulent environments, and the general signal processing problem of detecting intelligent messages in noisy data.

The tool we have developed is a frame-oriented discretized version of the continuous wavelet tranform W_F^ψ, where the wavelets ψ are specially constructed to detect periodic behavior in the signal F. We refer to such wavelets as piecewise constant wavelets of degree M; and in the case F is T-periodic, we compute that

$$a^{1/p} W_F^\psi(b,a) = \int_{\mathbf{R}} \psi\left(\frac{t-b}{a}\right) F(t)dt$$

Our first Theorem asserts that if F is T-periodic then there is an optimal piecewise constant wavelet ψ_0 and a point (b_0, a_0) in the time-scale plane such that

$$|W_F^{\psi_0}(b_0,a_0)| \geq |W_F^\psi(b,a)|$$

for all (b,a) in the time-scale plane and for all piecewise constant wavelets ψ of a given length.

This inequality is the basis of our periodicity detection algorithm, which also depends on a waveletgram averaging method. We have implemented our algorithm on synthesized data constructed to model periodic electrocorticogram seizure data. The algorithm successfully detects the periodicity of this data.

Because of the relevance of periodic wavelet transforms in the development of our periodicity detection algorithm, we have classified all wavelets ψ which give rise to such transforms. This classification is formulated in the following theorem, and the first sum in the expression for ψ in part 2 of the theorem is essentially a piecewise constant wavelet.

Theorem: *Let $\psi \in L^1(\mathbf{R})$. The following are equivalent:*

1. $a^{1/p} W_f^\psi(b,a) = \int_{\mathbf{R}} f(t)\psi(\frac{t-b}{a})dt$ *is 1-periodic in a for all $f \in L^\infty(\mathbf{T})$.*
2. $\widehat{\psi}(0) = 0$ *and ψ has the form:*

$$\psi(t) = \sum_{n\in\mathbf{Z}} a_n \mathbf{1}_{(n,n+1)}(t) + \sum_{n\in\mathbf{Z}} b_n \ln|t-n|$$

where $\{b_n\} \in l^1(\mathbf{Z})$, and $a_n = c_n + \pi i \sum_{k\leq n} b_k$ for some $\{c_n\} \in l^2(\mathbf{Z})$ for which $\{c_n - c_{n-1}\} \in l^1(\mathbf{Z})$.