MATHEMATICS 848G: INDEX THEORY ON MANIFOLDS
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This course will be a blend of analysis, geometry, and topology, perhaps even with applications to physics. The goal will be to explain the notion of *index* of a (linear) elliptic partial differential operator or, more generally, elliptic pseudodifferential operator, and to discuss the Atiyah-Singer Index Theorem and its variants and applications.

To put this in simple terms, index theory gives a way to use topology to count the number of solutions of a system of partial differential equations on a manifold, in the case where such a "count" makes sense. This has applications to many areas of mathematics, especially (complex) algebraic geometry, geometric analysis, and differential geometry and topology. It is also closely related to many important topics in functional analysis.

Covering the theory in complete detail from start to finish is probably too difficult, since too many different areas of mathematics are involved. However, students should have had basic graduate courses in at least one of analysis, geometry/topology, algebra, and modern physics, and should be prepared to read about some of the others.