Algebraic K-Theory and its Applications
by Jonathan Rosenberg

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Chapter I

page 11. In Exercise 1.2.9, part (2), the condition should read: ... all the entries of the matrix A are non-negative and no column of A is identically 0.

page 19, line 18. Change "sice" to "since".

page 25, line 3. Change "algebraic closure" to "integral closure".

page 26, bottom line. Change p to $\pm p$.

page 39, second paragraph. One small technical point: the monoid described in Exercise 1.1.7 really corresponds to the monoid of isomorphism classes of *oriented* vector bundles over S^2. The problem stems from the fact that $O(r)$ is disconnected and from the fact that $\pi_{n-1}(O(r))$ is defined using *based* maps from S^{n-1} to $O(r)$, which necessarily land in the connected component of the identity. This only makes a difference in the classification of rank-2 bundles, where the oriented bundles corresponding to integers m and $-m$ in $\mathbb{Z}$ are isomorphic as unoriented bundles. So to get the corresponding description of the monoid of unoriented bundles, one should replace the condition $m \in \mathbb{Z}$ by $m \in \mathbb{N}$ for $n = 2$.

Chapter II

page 65, lines 5 and 6 from bottom. This argument isn't correct when R is noncommutative, since in that case $(AB)^t$ may not be equal to $B^t A^t$, and in fact property (f) in the Theorem should have been restricted to the commutative case. Here is a counterexample with $R = \mathbb{H}$, the quaternions. Note that in this case $R^\times_{\text{ab}} = \mathbb{R}^\times_+$, the positive reals, since the group of unit quaternions (isomorphic to the Lie group $SU(2)$) is its own commutator subgroup. So the map $a \mapsto \bar{a}$ is just the usual absolute value for quaternions.

Let $n = 2$ and

$$A = \begin{pmatrix} 2 & i \\ j & k \end{pmatrix}, \quad \text{so} \quad A^t = \begin{pmatrix} 2 & j \\ i & k \end{pmatrix}.$$

Subtracting $\frac{j}{2}$ times the first row of A from the second row changes the second row to $(\,0 \quad k - \frac{j}{2}i\,)$ and thus results in a $\frac{3}{2}k$ in the lower right. So $\det A = 2 \cdot \frac{3}{2} = 3$. On the other hand, subtracting $\frac{i}{2}$ times the first row of A^t from the second row changes the second row to $(\,0 \quad k - \frac{i}{2}j\,)$ and thus results in a $\frac{1}{2}k$ in the lower right. So $\det A^t = 2 \cdot \frac{1}{2} = 1$ and $\det A \neq \det A^t$.

page 68, middle. The proof for the case $b_k \in R^\times$, $b_i \in \operatorname{rad} R$ for $i \neq k$, is incomplete. It should really read as follows:

In this case,

$$\det_n A' = (-1)^{k-1} \left(\overline{b'_k}\right)^{-1} \det_{n-1} \begin{pmatrix} B_1 \\ \vdots \\ B_i + aB_k \\ \vdots \\ \widehat{B_k} \\ \vdots \\ B_n \end{pmatrix},$$

and

$$\det_n A = (-1)^{k-1} \bar{b}_k^{-1} \det_{n-1} \begin{pmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ \widehat{B_k} \\ \vdots \\ B_n \end{pmatrix}.$$

But

$$B_i + aB_k = B_i - a \sum_{j \neq k} b_k^{-1} b_j B_j = (1 - ab_k^{-1} b_i) B_i - a \sum_{j \neq i,k} b_k^{-1} b_j B_j,$$

so by properties (a) and (c) for $\det_{n-1}$, we have

$$\det_n A' = (-1)^{k-1} \left(\overline{b'_k}\right)^{-1} \left(\overline{1 - ab_k^{-1} b_i}\right) \det_{n-1} \begin{pmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ \widehat{B_k} \\ \vdots \\ B_n \end{pmatrix}.$$

Thus to show that $\det_n A' = \det_n A$, it suffices to show that

$$\left(\overline{b_k - b_i a}\right)^{-1} \left(\overline{1 - ab_k^{-1} b_i}\right) = \bar{b}_k^{-1}$$

in $R^{\times}_{\mathrm{ab}}$, or that

$$(1 - ab_k^{-1} b_i) b_k \equiv b_k - b_i a \ \text{ in } R^{\times} \bmod \left[R^{\times}, R^{\times}\right].$$

If we factor the right-hand side as $b_k(1 - b_k^{-1} b_i a)$ and let $u = b_k^{-1} b_i \in \operatorname{rad} R$, then it suffices to show that

$$1 - au \equiv 1 - ua \ \text{ in } R^{\times} \bmod \left[R^{\times}, R^{\times}\right].$$

There are now two cases. If $a \in R^\times$, then $1 - au = a(a^{-1} - u)$ while $1 - ua = (a^{-1} - u)a$, and $(a^{-1} - u) \in R^\times$ since $u \in \operatorname{rad} R$. So this case is clear. If, on the other hand, $a \notin R^\times$, then $a \in \operatorname{rad} R$, so $(1 - a) \in R^\times$, $v = 1 + u + ua \in R^\times$, and

$$\begin{aligned}
(1 - au)(1 - a) &= (1 - a - au + aua) = 1 - av, \\
(1 - ua)(1 - a) &= (1 - a - ua + ua^2) = 1 - va, \\
\text{so } (1 - au)(1 - ua)^{-1} &= (1 - av)(1 - va)^{-1} \\
&= (v^{-1} - a)v(v^{-1} - a)^{-1}v^{-1} \in \left[R^\times, R^\times\right].
\end{aligned}$$

So this confirms property (a) for $\det_n$.

page 69, bottom line. The last sentence should say: Deduce that $e^{2j\theta_1}$, $e^{2k\theta_2}$, and $e^{2i\theta_3}$ are all commutators.

page 76, line 10. Change “One” to “Once”.

page 80, line 6. Change $\mathbb{R}^{r_1+r+2}$ to $\mathbb{R}^{r_1+r_2}$.

page 82, line 15. Change E_1 to E_2.

page 91, third line from bottom. This should read: Suppose (C_1, d_1) and (C_2, d_2) are complexes … .

page 104, line 6. The displayed equation should read

$$[a\ b]_I^k = [a\ b^k]_I = 1.$$

Chapter III

page 149, 9th line from bottom. The equation should read $A'T = TA$, not $A'T = TP$.

pages 150–151. There are some problems with the argument given here, which are pointed out and discussed in “Strong shift equivalence and algebraic K-theory” by Mike Boyle and Scott Schmieding, arXiv:1501.04695, Remark 8.2. These authors suggest a way around the problem.

page 158. In Exercise 3.3.6, the correct statement of part (1) is as follows: Fill in the details of the argument copied from Theorem 1.3.11, that if S is a ring and if J is an ideal of S contained in $\operatorname{rad} S$, then the map $K_0(S) \to K_0(S/J)$ induced by the quotient map $S \to S/J$ is injective.

Chapter IV

page 184. The matrix identity in part (1) of Exercise 4.1.28 was supposed to read:

$$\begin{pmatrix} d & 0 \\ 0 & d^{-1} \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d^{-1} & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} 1 & -a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & (d^2 - 1)a \\ 0 & 1 \end{pmatrix}$$

page 189. In the proof of Theorem 4.2.4, there is a sentence missing two lines up from the end of the proof. Before the sentence "Since these generate ..." one should insert the line: Similarly, x commutes with $x_{Nk}(a)$ for N large enough, for any $k < N$, and for any $a \in R$.

page 190. The beginning of the proof of Theorem 4.2.7 should say: "By Theorem 4.2.4".

page 191, line 5 from the bottom. This should read $s : \mathrm{St}(R) \to U$.

page 237. There are some misprints in the last sentence of the statement of Theorem 4.4.19. The sentence should read: "Similarly, if $\mathbb{F} = \mathbb{C}$, $K_0(R) \cong KU^0(X)$, $K_1(R)$ is an extension of $KU^{-1}(X)$ by the connected component of the identity in $R^\times$, and $K_2(R)$ surjects onto $\widetilde{KU}^{-2}(X)$."

page 244, lines 6 and 7 from the bottom. The phrase "with the map $K_1(R[s, t], (st-1)) \to K_1(R)$ induced by mapping $R[s, t]$ to R" should be replaced by "with a certain map $K_1(R[s, t], (st-1)) \to K_1(R)$". (The obvious map is 0, but there is another map that can be defined with more work.)

Chapter V

page 245, one line up from bottom. The sequence should read:

$$\cdots \to K_{i+1}(R/I) \xrightarrow{\partial} K_i(R, I) \to K_i(R) \to K_i(R/I) \xrightarrow{\partial} K_{i-1}(R, I) \to \dots$$

page 258. The statement of part (1) of Proposition 5.1.20 should say: The projection $p : E = B \times F \to B$ onto the first factor ....

page 266, line 14. The displayed equation should read:

$$H_\bullet(\tilde{X}^+, \tilde{X}; \mathbb{Z}) = 0.$$

Chapter VI

page 322. line 2. $HP^\bullet(R)$ should be $HC^\bullet(R)$.

pages 322–325. The history of these Morita invariance results is somewhat complicated, and the main ideas are due to K. Dennis and A. Connes, but it should have been mentioned that the proof given here is due to R. McCarthy, "Morita equivalence and cyclic homology," *C. R. Acad. Sci. Paris, Sér. I Math.* **307** (1988), 211–215.

page 331. The formula in Exercise 6.1.48 should read:

$$HH_n(R[u, u^{-1}]) \cong HH_n(R)[u, u^{-1}] \oplus HH_{n-1}(R)[u, u^{-1}].$$

page 352. It should have been assumed in Definition 6.3.3 that R is finitely generated as an abelian group, or at least that all torsion is of order prime to n. Otherwise, the indicated sequences of chain complexes may not be short exact. One can get around this in general by replacing $C_\bullet(R) \otimes_{\mathbb{Z}} \mathbb{Z}/n$ with the mapping cone of the map $C_\bullet(R) \xrightarrow{n} C_\bullet(R)$. Fortunately, there is no need to do this for the applications in 6.3.4 and 6.3.5.

page 364. It should have been stated in Definition 6.3.21 that R must be commutative.