Solutions to some Exercises

Solutions for HW 1:

Exercise 124 A $(w, x, y, z) = (10, 0, -1, 0) + x(-1, 1, 0, 0) + z(-2, 0, 1, 1), \forall_x$ and $_z$.

Exercise 124 B $(w, x, y, z) = 100(10, 0, -1, 0) + x(-1, 1, 0, 0) + z(-2, 0, 1, 1), \forall_x$ and $_z$.

Exercise 1.38. Part I: $(u, v, w, x, y, z) = (0, 0, 0, 0, 10, 0)$

II $(u, v, w, x, y, z) = (16, 8, 4, 2, 11, 1)$

Exercise 1.39. All $u, v, w, x, y, z = 1.1$, but changing one number by 10% yields: $(u, v, w, x, y, z) = (-.5, .3, .7, .9, 1, 1)$.

Exercise1.52 $(x, y, z) = (-2, 0, -1)$ and $(1/3)(-1, 5, 2)$

Exercise 1.55 $(u, v, w, x, y, z) = (-4, -2, -1, 0, 0, 0) + x(4, 2, 1, 1, 0, 0) + y(4, 2, 1, 0, 1, 0) + z(4, 2, 1, 0, 0, 1), \forall_{x,y}$ and $_z$.

Exercise I.4.40.

Here is a simpler proof than the one given in class:

Exercise I.4.40. *Given a point r on a line l thru the origin in the plane (or n-space, R^n) and a point p. Find a formula for the perpendicular projection of point p into line l, in terms of p and r.*

Let q be the perpendicular projection of point p into the line l.

The geometry of the situation has two pieces:

(*) The point $\mathbf{q}$ is on the line through the origin, $(\mathbf{0})$ and the point $\mathbf{r}$. Hence, the point $\mathbf{q} = A\mathbf{r}$, for an unknown, to-be-found number, A.

(*) Then the geometric vector $\overline{pq}$ is perpendicular to the geometric vector $\overline{oq}$ and hence $\overline{pq}$ is also perpendicular to the geometric vector $\overline{or}$, extended.

(*) $\overline{pq} \perp \overline{or} \iff 0 = \overline{pq} \cdot \overline{or} = (\overline{oq} - \overline{op}) \cdot \overline{or}$. Since the arithmetic for geometric vectors and for coordinate vectors mimic each other:

$$0 = (\mathbf{q} - \mathbf{p}) \cdot \mathbf{r} = (A\mathbf{r} - \mathbf{p}) \cdot \mathbf{r} = A\mathbf{r} \cdot \mathbf{r} - \mathbf{p} \cdot \mathbf{r}.$$

Solve for A: $A = \frac{\mathbf{p} \cdot \mathbf{r}}{\mathbf{r} \cdot \mathbf{r}}$. Hence: $\boxed{\mathbf{q} = A\mathbf{r} = \frac{\mathbf{p} \cdot \mathbf{r}}{\mathbf{r} \cdot \mathbf{r}}\mathbf{r}.}$

Quiz on Linear Equations with Answers

(a) State the defining equations for linear transformations.

(i) $L(u+v) = L(u) + L(v)$, $\forall$ vectors u and v.

(ii) $L(Av) = AL(v)$, $\forall$ vectors v and $\forall$ numbers A.

or combine these two equations as:

$L(Au + Bv) = AL(u) + BL(v)$, $\forall$ vectors v and w and $\forall$ numbers A and B.

(b) State and prove the theorem on homogeneous linear equations.

This is Theorem II.3.1. There are two statements of this theorem in the text.

Proof by calculation: Given $L(v) = 0$ and $L(w) = 0$ and $u = Av + Bw$.

Then $L(Av+Bw) = AL(v)+BL(w) = A\times 0+B\times 0 = 0$. Thus $L(u) = 0$, $\forall_A$ and B. $\surd$

(c) State the algorithm implied by the theorem on homogeneous linear equations. From text, immediately after Theorem II.3.1:

Step 0. Observe/Check that the equation is indeed a homogeneous linear equation

Step 1. Find the simplest solutions.

Step 2. Take all their linear combinations.

(d) State the theorem on non-homogeneous linear equations. This is Theorem II.4.3.

(e) Given $\ddot{x} - x = 3$.

(i) Using the Proposition on Recognizing linear transformation, prove that the relevant transformation is a linear transformation. State which parts of the Proposition on Recognizing linear transformation you used.

Set $L(x) = \ddot{x} - x = 3$. Review Example II.2.5, use either scheme.

(ii) Solve the equation. Indicate the line where the theorem on homogeneous linear equations is used. Indicate the line where the theorem on non-homogeneous linear equations is used.

Similar to Example II.4.8 and Example II.3.7.

For a simple solution to the non-homogeneous linear equation, try $x_c = C$, where C is an unknown, to-be-found constant. Then $L(C) = 0 - C = 3$, $\implies$ $x_c = C = -3$. The associated homogeneous equation is $L(u) = \ddot{u} - u = 0$. Try $u = e^{rt}$, where r is an unknown, to-be-found constant. Plug in, solve for $r = \pm 1$. Hence $u_1 = e^t$ and $u_2 = e^{-t}$ are two solutions of the associated homogeneous equation.

Using the theorem on homogeneous linear equations: $u = Au_1 + Bu_2 = Ae^t + Be^{-t}, \forall_A$ and $_B$.

Using the theorem on non-homogeneous linear equations: $x = -3 + Ae^t + Be^{-t}, \forall_A$ and $_B$.

Answers to some exercises in Ch II Sec. 4.

Exercise II.4.8 The differential equation is similar to the ones solved in Examples II.4.4 and II.4.7.

The equation may be rewritten in linear equation form as: $L(x) = \dot{x} - .001x = 7$. We recognize this transformation, $L(u)$ as a linear transformation (Example II.2.5). Hence the equation is a linear equation.

The associated homogeneous equation is $L(u) = \dot{u} - .001u = 0$, which may be rewritten as: $\dot{u} = .001u$. This is an "exponential growth" equation. (Appendix C Section 2 or your calculus textbook) The solution to $L(u) = \dot{u} - .001u = 0$ is $\boxed{u(t) = Ae^{\frac{t}{1000}}, \quad \forall_A.}$

To find a simple solution to the non-homogeneous linear equation, $L(x) = 7$, we note that the output is a constant, so an input of a constant usually works. Try $x_c = C$, where C is an unknown, to-be-found constant. Then $L(C) = 0 - .001C = 7$. Hence $\boxed{x_c = C = -7000}$ is a simple solution. YEA.

Now, the Theorem II.4.3 on non-homogeneous equations provides that the solution to $L(x) = \dot{x} - .001x = 7$ is $\boxed{x(t) = -7000 - Ae^{\frac{t}{1000}}, \quad \forall_A.}$

For $-7000 = x_1(0) = -7000 - Ae^0 = -7000 - A$. Hence, $A = -1$ and $x(t) = -7000 - e^{\frac{t}{1000}}$. Then $\lim_{t \longrightarrow \infty} x_1(t) = -\infty$.

When $x_2(0) = -6999$, $A = +1$ and $x(t) = -7000 + e^{\frac{t}{1000}}$.

Hence $\lim_{t \longrightarrow \infty} x_2(t) = +\infty$.

A seemingly small difference in the initial condition results massive difference in long term behavior. It takes a while for the $e^{\frac{t}{1000}}$ to become large, but it does. The differential equation was called **unstable** in the differential equation course.

Exercise II.4.9 The differential equation is the same as the one solved in Example II.4.4

Exercise II.4.11 The differential equation is similsr to the one solved in Example II.4.9.

For $\ddot{x} + 3x = \sin\ 2t$, a particular solution is x(t) = -sin $2t$.

For $\ddot{x} + 3x = \sin\ 1.7t$, a particular solution is x(t) $\approx 9\ \sin\ 1.7t$.

For $\ddot{x} + 3x = \sin\ 1.732t$, a particular solution is x(t) $\approx 5681\ \sin\ 1.732t$.

Those who round $(1.732)^2$ to 2.99 or 2.999 will get very different answers.

For $\ddot{x} + 3x = \sin\ \omega t$, a particular solution is x(t) $\approx \frac{1}{3-\omega^2}\ \sin\ 1.732t$. As ω^2 gets close to 3, the coefficient of the sine term goes to infinity. This is related to resonnance and beats, which is discussed in textbooks on differential equations.

Exercise III.2.14. Shortcuts: Use a big zero instead of writing a block of many zeros.

Observe that $C = 10B$, then $C^2 = 100B^2$ and $C^3 = 1000C^3$, etc.

$\boxed{B^6 = O = C^6.}$

Exercise III.2.17. An answer is supplied by Proposition III.5.9.

Exercise III.2.31. Note: The last matrix is a 1×1-mx; not a 2×1-mx. One tip-off is the plus sign at the end of the top line. The other tip-off is that multiplying $(x, y, z)S \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ produces a 1×1-mx.

Exercise III.2.32. An answer is supplied in Ch. 7 Sec.2 by Observation 2.3 (turn page) and by the first 4 lines of Corollary 7.2.3 (turn page).

Exercise III.2.37. The purpose of this exercise is to find "which" matrices commute with the diagonal matrix: $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. In math, the word "which" means "all those which".

The set of matrices, which commute with $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ is the set of *all* 3×3-diagonal matrices. That is, if $MD = DM$, then M is a 3×3-diagonal matrix.

(iii) A general rule is that, if $D = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$, then $D^n = \begin{pmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{pmatrix}$, $\forall_{n \in Z^+}$.
For $n = 0$, one uses the definition: $M^0 = I_n$, $\forall_{n \times n-matrices}$. This formula is also valid for $n = -1$.

Exercise III.2.37. The matrices, which commute with $D_2 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ are the matrices, $\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{pmatrix}$, $\forall_{a,b,c,d,e}$.

Exercise III.3.7. **(c)** By part (b), $N = PMP^{-1}$, is the product of invertible matrices, hence it is also invertible. (General product Rule for Inverses.) Then using the formula for the inverse of the product of invertible matrices, $((ABC)^{-1} = C^{-1}B^{-1}A^{-1})$,

$$N = PMP^{-1} \Longrightarrow N^{-1} = (P^{-1})^{-1}M^{-1}P^{-1} = PM^{-1}P^{-1}$$

Similarly, given $M = P^{-1}NP$, using the formula for the inverse of the product of invertible matrices, will provide $M^{-1} = P^{-1}N^{-1}P$.

Exercise III.4.9. The stochastic matrix is $\begin{pmatrix} .5 & .7 \\ .5 & .3 \end{pmatrix}$. Tomorrow's probability vector is

$\binom{.56}{.44}$. Yesterday's probability vector was $\binom{0}{1}$.

Exercise III.4.12. After doing the matrix multiplication, both off diagonal terms are:

$$-b(\sin^2\theta - \cos^2\theta) + (c-a)\sin\theta\cos\theta = 0.$$

Either (i) use the double angle formulas for $\sin 2\theta$ and $\cos 2\theta$, and then solve for $\tan 2\theta$, or (ii) divide by $\sin\theta\cos\theta$, convert into a quadratic equation in $\tan\theta$, set $x = \tan\theta$, solve for x.

When $a = c$, off diagonal terms simplify to $-b(\sin^2\theta - \cos^2\theta) = 0$, which implies that $\theta = \frac{\pi}{4}$.

Exercise III.4.24 and 17. See the proof of Proposition VI.3.2.

Rule III.6.4 (i) To prove $(A^{-1})^T = (A^T)^{-1}$

Remark. We will use the defining equation for inverse matrices, which says that two matrices, M and N are inverse matrices, when $MN = I = NM$. Or verbally, two matrices are inverse matrices, when their product (both ways) is the identity matrix.

We will use the Product Rule for transposes.

Proof. #1. Since A is an invertible matrix:

$$AA^{-1} = I = A^{-1}A$$

$$I = I^T = (A^{-1}A)^T = A^T\ (A^{-1})^T$$

Similarly:

$$I = I^T = (AA^{-1})^T = (A^{-1})^T\ A^T.$$

Thus the product (both ways) of $(A^{-1})^T$ and (A^T) is the identity matrix; hence

$$\boxed{(A^{-1})^T = (A^T)^{-1}.}$$

Proof. #2. The equation, $(A^{-1})^T = (A^T)^{-1}$ means that $(A^{-1})^T$ and (A^T) are inverse matrices. To prove this, we check that their product (both ways) is the identity matrix:

$$(A^{-1})^T \times (A^T) = (A\ A^{-1})^T = I^T = I$$

$$(A^T) \times (A^{-1})^T = (A^{-1}\ A)^T = I^T = I$$

hence $\boxed{(A^{-1})^T = (A^T)^{-1}.}$

Exercise III.6.14 (b) and key part of proof of Theorem III.7.5.

$$P = M(M^TM)^{-1}M^T$$

$$P^T = [M \times (M^TM)^{-1} \times M^T]^T$$

What is this? This is the transpose of the product of matrices. So we use the Product Rule for transposes. Thus:

$$P^T = [M^T]^T \times [(M^T M)^{-1}]^T \times M^T = M \times \left[(M^T M)^T\right]^{-1} \times M^T$$

$$(M^T M)^T = M^T [M^T]^T = M^T M$$

We have also used the rules $[M^T]^T = M$ and $(A^{-1})^T = (A^T)^{-1}$.

Exercise III.6A.8. Let S be an invertible symmetric matrix. Check that S^{-1} is also symmetric.

Proof.

$$(S^{-1})^T = (S^T)^{-1} = S^{-1},$$

(using the "inverse" rule for tranposes.) Thus S^{-1} is also symmetric.

Exercise III.6A.11. Given unknown 7×7 matrices, S, D and P, where P is an invertible matrix and S is a symmetric matrix, such that $S = P^T D P$. Let $\mathbf{v} \in R^7$ be a coordinate vector. Let $\mathbf{w} = P\mathbf{v}$. Given $(z) = \mathbf{v}^T S \mathbf{v}$.

Show that $(z) = \mathbf{w}^T D \mathbf{w}$.

Proof.

$$(z) = \mathbf{v}^T S \mathbf{v} = \mathbf{v}^T P^T D P \mathbf{v} = \mathbf{w}^T D \mathbf{w}$$

since $\mathbf{v}^T P^T = (P\mathbf{v})^T = \mathbf{w}$.

Answers to some exercises for Ch. IV.

Exercise IV.3A.2. $|E| \le J_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. $|M^{-1}E|\mathbf{1} \le |M^{-1}| \times |E|\mathbf{1} \le \frac{1}{49}\binom{22}{18} < \mathbf{1}$. Therefore, $M+E$ is an invertible matrix.

Exercise IV.3A.3. $|M^{-1}E|\mathbf{1} \le |M^{-1}| \times |E|\mathbf{1} \le \binom{2.2}{1.8} > \mathbf{1}$. Therefore, the lemma provides *no* info on the invertibility of the matrix, $M+E$.

Remember, E is unknown, hence $M^{-1}E$ is unknown and cannot be calculated. Only, bounds on E are known. Therefore, need to use $|M^{-1}E|\mathbf{1} \le |M^{-1}| \times |E|\mathbf{1}$.

Using $E = \frac{1}{10}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, and calculating $|M^{-1}E|\mathbf{1} = \left| M^{-1}\frac{1}{10}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right| \mathbf{1}$ will produce a *wrong* answer.

Exercise IV.3B.2. $d = 0.4002 < 1$. Hence $M+E$ is invertible. $||\Delta v||_\infty \le 1.6681$.

Exercise IV.3B.3. $d = 0.1 < 1$. Hence $M+E$ is invertible.

Part (i) According to Theorem IV.3B.1, $\Delta v = 0$; according to Corollary IV.3B.2, $||\Delta v||_\infty \le \frac{1}{9}$. For this matrix and vector, the error bound provided by the theorem is much lower than the one provided by the corollary.

Part (ii) According to both Theorem IV.3B.1 and Corollary IV.3B.2, $||\Delta v||_\infty \le \frac{1}{9}$. For this matrix and vector, the two error bounds are the same.

Exercise IV.3B.5. (Not Superposition)

Part (a). $|\Delta v_a| \le \begin{pmatrix} .004 \\ .006 \end{pmatrix}$.

Part (b). $d = 0.012 < 1$. Hence $M+E$ is invertible. $||\Delta v_b||_\infty \le \frac{.066}{1-0.12} = 0.0668$.

Part (c). $||\Delta v_c||_\infty \le 0.0668 + \frac{.006}{1-0.12} = 0.07287$.

Part (d). $||\Delta v_a||_\infty + ||\Delta v_b||_\infty \le 0.006 + 0.0668 = 0.0724 < 0.07287$, the bound for $||\Delta v_c||_\infty$.

Thus the error bound, due to the two errors, is larger than the sum of the two error bounds due to the individual errors. This is to be expected, since the total error due to two individual errors is often larger than the sum. Superposition does *not* occur for errors or their bounds.

Answers to some exercises for Ch. VI Sec. 1.

Exercise VI.1.5 (a) (i)

Given $M = \begin{pmatrix} -1 & 8 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{pmatrix}$.

To find the eigenvalues: Solve the characteristic equation, using Formula 5.6 (iii) for block triangular determinants:

$$\det(M{-}rI) = \begin{vmatrix} -1-r & 8 & 0 \\ 2 & -1-r & 0 \\ 0 & 0 & 7-r \end{vmatrix} = \begin{vmatrix} -1-r & 8 \\ 2 & -1-r \end{vmatrix}(7{-}r) = [(-1{-}r)^2{-}16](7{-}r).$$

$$0 = \det(M{-}rI) = [(-1{-}r)^2{-}16](7{-}r) \Longrightarrow [(-1{-}r)^2{-}16] = 0 = (7{-}r) \Longrightarrow r = 7 \text{ and } (-1{-}r)^2 = 16.$$

$$(-1-r)^2 = 16 \Longrightarrow -1-r = \pm 4 \Longrightarrow r = -1 \pm 4 = -5, 3$$

Thus the eigenvalues are $r = -5, 3, 7$.

For the eigenvalue $r = -5$, the eigenspace is $\mathit{Span}\{(-2, 1, 0)^T\}$.

For the eigenvalue $r = 3$, the eigenspace is $\mathit{Span}\{(2, 1, 0)^T\}$.

For the eigenvalue $r = 7$, the eigenspace is $\mathit{Span}\{(0, 0, 1)^T\}$, that is the z-axis.

(b) For a block diagonal matrix, $M = \begin{vmatrix} A & O \\ O & B \end{vmatrix}$, the eigenvalues of M are the eigenvalues of A and B. The eigenvectors of M are the eigenvectors of A and B, with zeros "added" for the extra coordinates.

Exercise VI.1.6 (a) To find the eigenvalues: Solve the characteristic equation, using Formula 5.6 (ii) for triangular determinants, which says to just multiply down the main diagonal.

$$0 = \det(M - rI) = (1-r)(2-r)\cdots(7-r) \Longrightarrow r = 1, 2, \cdots 7.$$

(b) For a triangular matrix, T, the eigenvalues of T are the numbers on the main diagonal.

(c) The eigenvalues are $r = 4 \pm \sqrt{6}$. This matrix is **not** a triangular matrix, so the rule of Part (b) does **not** apply.

Exercise VI.1.9 The eigendata for these matrices are calculated in the manner of Example III.3.7.

(a) For the matrix J_3; its eigenvalues are $r = 3$ and 0.

For eigenvalue $r = 3$, the eigenspace is $\mathit{Span}\{(1, 1, 1)^T\}$.

For eigenvalue $r = 0$, the eigenspace is $\mathit{Span}\{(1, 0, -1)^T, (1, -1, 0)^T\}$. A basis for this eigenspace is $\{(1, 0, -1)^T, (1, -1, 0)^T\}$, and the $\dim \mathit{eigenspace} = 2$. There are many other bases for this eigenspace.

(d) The matrix is $J_3 + 6I_3$; its eigenvalues are $r = 9$ and 6.

For eigenvalue $r = 9$, the eigenspace is $Span\{(1,1,1)^T\}$.

For eigenvalue $r = 6$, the eigenspace is $Span\{(1,0,-1)^T, (1,-1,0)^T\}$.

The eigenspaces for the matrices, J_3 and $J_3 + 6I_3$ are the same.

(b) The eigenvalues are $r = 1, -1$ and 0.

For eigenvalue $r = 1$, the eigenspace is $Span\{(1,1,1)^T\}$.

For eigenvalue $r = -1$, the eigenspace is $Span\{(-1,1,1)^T\}$.

For eigenvalue $r = 0$, the eigenspace is $Span\{(0,1,-1)^T\}$.

(c) The eigenvalues are $r = 1, -1$ and 0.

For eigenvalue $r = 1$, the eigenspace is $Span\{(1,1,1)^T\}$.

For eigenvalue $r = -1$, the eigenspace is $Span\{(-1,1,1)^T\}$.

For eigenvalue $r = 0$, the eigenspace is $Span\{(0,2,-1)^T\}$.

Warning. Gaussing a matrix will usually **change** the eigenvalues. Therefore, Gaussing a matrix will usually result in **wrong** eigenvalues.

Exercise VI.1.22. Warning, these matrices are not triangular. Do not look for a shortcut.

Exercise VI.1.24. Writing $\dot{x} = 2x$ and $\dot{y} = 3y$ in matrix-vector notation:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Set $v = \begin{pmatrix} x \\ y \end{pmatrix}$, then $\dot{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$. Set $M = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, now the matrix-vector equation becomes: $\dot{v} = Mv$.

The eigenvalues of this diagonal matrix, M are the numbers on the main diagonal: $r = 2$ and 3.

For eigenvalue $r = 2$, the eigenspace is $Span\{(1,0)^T\}$, the x-axis.

For eigenvalue $r = 3$, the eigenspace is $Span\{(0,1)^T\}$, the y-axis.

Eigenvalue-eigenvector solutions to $\dot{v} = Mv$ are $v_1 = e^{2t}\binom{1}{0}$ and $v_2 = e^{3t}\binom{0}{1}$.

Since $\dot{v} = Mv$ is [equivalent to] a homogeneous linear equation, all linear combo's of solutions are more solutions. Thus

$$\begin{pmatrix} x \\ y \end{pmatrix} = v = Av_1 + Bv_2 = Ae^{2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + Be^{3t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ \forall_A \text{ and } _B.$$

and

$$x = Ae^{2t} \text{ and } y = +Be^{3t},\ \forall_A \text{ and }_B.$$

Remark. Of course, $\dot{x} = 2x$ and $\dot{y} = 3y$ are both exponential growth equations. For this reason, their solutions are $x = Ae^{2t}$ and $y = +Be^{3t}$, $\forall_A$ and $_B$. Nice to see that the matrix method of solving these two equations yields the same answers.

Remark. Now, you may calculate that:

$$A^3y^2 = B^2x^3,\ \forall_A \text{ and }_B,$$

Except when $A = 0$, we see that $y = Cx^{\frac{3}{2}}$, $\forall_C$, with $C = \frac{B}{A^{1.5}}$. Thus, all these solution curves have "parabola-type" shapes and are tangent to the x-axis at the origin, except when $A = 0$.

Exercise VI.1.25. Writing $\dot{x} = 9y$ and $\dot{y} = x$ in matrix-vector notation:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & 9 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Set $v = \begin{pmatrix} x \\ y \end{pmatrix}$, then $\dot{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$. Set $M = \begin{pmatrix} 0 & 9 \\ 1 & 0 \end{pmatrix}$, now the matrix-vector becomes: $\dot{v} = Mv$.

The eigenvalues of matrix, M are $r = \pm 3$.

For eigenvalue $r = 3$, the eigenspace is $Span\{(3,1)^T\}$.

For eigenvalue $r = -3$, the eigenspace is $Span\{(-3,1)^T\}$.

Eigenvalue-eigenvector solutions to $\dot{v} = Mv$ are $v_1 = e^{3t}\binom{3}{1}$ and $v_2 = e^{-3t}\binom{-3}{1}$.

Since $\dot{v} = Mv$ is [equivalent to] a homogeneous linear equation, all linear combo's of solutions are more solutions. Thus

$$\begin{pmatrix} x \\ y \end{pmatrix} = v = Av_1 + Bv_2 = Ae^{3t}\begin{pmatrix} 3 \\ 1 \end{pmatrix} + Be^{-3t}\begin{pmatrix} -3 \\ 1 \end{pmatrix},\ \forall_A \text{ and }_B.$$

and

$$x = 3Ae^{3t} - 3Be^{-3t} \text{ and } y = Ae^{3t} + Be^{-3t},\ \forall_A \text{ and }_B.$$

Now, you may calculate that:

$$-x^2 + 9y^2 = -(3Ae^{3t} - 3Be^{-3t})^2 + 9(Ae^{3t} + Be^{-3t})^2 = 36AB,\ \forall_A \text{ and }_B.$$

Thus, all these curves are hyperboli, with the lines $3y = \pm x$ as the asymptotes.