AMSC 466. Spring 2020

Homework Assignment 10. Due Thursday April 30.

1. **(5 pts)** Derive the one-sided backward difference second-order estimator for the first derivative
$$D^1_{2-}[f](x,h) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}. \quad (1)$$
and an error estimate for it using the method of undetermined coefficients. I.e., write
$$D^1_{2-}[f](x,h) = \frac{1}{h}\left[af(x) + bf(x-h) + cf(h-2h)\right] = f'(x) + Ch^2 + \ldots,$$
Taylor-expand the terms $f(x-h)$ and $f(x-2h)$ around x, and set up and solve an appropriate linear system for the coefficients a, b, and c. As a result, you will be also find the constant C in the main error term.

2. (a) **(5 pts)** Use one step of Richardson extrapolation to derive (1). Proceed as follows. Start with the first-order backward difference estimator:
$$F(h) := \frac{f(x) - f(x-h)}{h}.$$
Taylor-expand $f(x-h)$ around x and obtain a series:
$$F(h) = f'(x) + a_1 h + a_2 h^2 + a_3 h^3 \ldots.$$
Then write out this estimator for h and $2h$. Use a linear combination of $F(h)$ and $F(2h)$ to knock out the error term proportional to h. Obtain a second-order estimator for f' and an error formula for it. This estimator must coincide with (1).

   (b) **(5 pts)** Denote the second-order estimator obtained in the previous item by $F(h, 2h)$. Use F with $2h$ and $3h$ to obtain another second-order estimator $F(2h, 3h)$. Use an appropriate linear combination of $F(h, 2h)$ and $F(2h, 3h)$ to get a third-order estimator for f'. Write out the resulting estimator in terms of $f(x)$, $f(x-h)$, $f(x-2h)$, and $f(x-3h)$. Check your result using the table "Backward finite difference" in Wiki: finite difference coefficient.

3. **(5 pts)** Write out the Lagrange polynomial $p(x)$ interpolating a smooth function $f(x)$ at $x-h$, x, and $x+h$. Take the derivative of $p(x)$ with respect to x and evaluate it at $x-h$, x, and $x+h$. Compare your results with second-order backward-, central-, and forward-difference estimators for the first derivative of f. Comment on your results.