AMSC 466. Spring 2020

Homework Assignment 11. Due Thursday May 7.

1. **(5 pts)** Let f be four times continuously differentiable on $[a, b]$. Derive Simpson's quadrature rule and the error estimate for it:

$$I(f) = \int_a^b f(x)dx = S(f) + E^S(f)$$

where

$$S(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right],$$

$$E^S = -\frac{1}{90}f^{(iv)}(\eta)\left(\frac{b-a}{2}\right)^5.$$

2. **(5 pts)** Consider an adaptive quadrature method based on the trapezoidal rule where each interval is divided into 3 equal subintervals whenever the error estimate exceeds the tolerance. Derive and justify an error estimate for this method of the form

$$E = \alpha|T(a,c) + T(c,d) + T(d,b) - T(a,b)|,$$

with

$$T(x,y) = \tfrac{1}{2}(y-x)[f(x) + f(y)],$$

where $c = a + \frac{1}{3}(b-a)$, $d = a + \frac{2}{3}(b-a)$, and α are to be found.

Hint: mimic the argument in `quadrature.pdf`.

3. **(5 pts)** Write a program implementing the adaptive trapezoidal rule that you have developed in the previous problem. Use it to integrate $f(x) = (1+x^2)^{-1}$ over the interval $[-5, 5]$. Make a table showing the number of correct digits and the number of nodes for various values of tolerance. *Hint: modify my code* `AdaptiveSimpson.m` *or the Python code in* Wiki: Adaptive Simpson's.