Final exam. Problem 2.

This problem requires both analytical calculations and numerical experiments. Submit a pdf file with your report and figures and link your codes to it.

1. Consider a SIR model for propagating infectious disease over a random graph with degree distribution $\{p(k)\}$ such that there exists a giant component. Let T be the probability for any given edge to transmit the infection. What is the critical transmissibility T_c such that if $T < T_c$, epidemic cannot occur, while if $T > T_c$, it may occur? Express it via $\kappa := \langle k^2 \rangle / \langle k \rangle$, the ratio of the second and the first moments in the original graph.

   Assume that $T > T_c$, i.e., an epidemic is possible. Suppose that we vaccinate a fraction v of randomly selected nodes. Derive an expression for the critical fraction of nodes v_c such that if $v > v_c$, epidemic cannot occur, while if $v < v_c$, epidemic may occur.

2. Consider an infinitely large graph with power-law degree distribution:

$$p(k) = \frac{k^{-\alpha}}{\zeta(\alpha)}, \quad k = 1, 2, \ldots, \quad \text{where} \quad \zeta(\alpha) := \sum_{k=1}^{\infty} k^{-\alpha}$$

   is the Riemann zeta-function. Suppose $\alpha = 2.2$. Find the numerical values for:

   (a) The fraction of nodes in the giant component.

   (b) For transmissibility $T = 0.4$, the fraction of nodes affected by the epidemic if it occurs.

   (c) The critical fraction v_c to vaccinate in order to eliminate the possibility of epidemic.

3. Generate a random graph with $n = 10^4$ nodes and power-law degree distribution with $\alpha = 2.2$. You are welcome to use my routine

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[G,edges,K,p] = MakePowerLawRandomGraph(n,a).
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   This routine implements the procedure described here. Please feel free to modify it or write your own routine.

   For the generated finite graphs, find:

   (a) The average fraction of vertices in the giant component.

   (b) The average fraction of nodes affected by an epidemic for $T = 0.4$.

   (c) The critical value T_c (it will be different from the one for an infinite graph due to the finite-size effect).

   (d) The critical fraction v_c to vaccinate for $T = 0.4$.

4. Run a discrete-time SIR model on a graph from the previous item starting from a single infecting node. Assume that each infecting node remains infecting for one time step. Plot the fraction of infecting nodes vs time. Repeat 100 times. Make a prediction for how the duration of the epidemic scales with the number of nodes in the graph. What is the relationship between this SIR model and the BFS?