AMSC808N/CMSC828V Maria Cameron

Homework 2. Due Thursday, Sept. 24

The first three problems are the exercises from Section 4 "Duality" in `2-OptProb&Methods4Classification.pdf`. Problem 4 is an exercise to design a nonlinear mapping to a feature space.

All equation, figure, and section numbers refer to `2-OptProb&Methods4Classification.pdf`.

1. **(5 pts)** Is the matrix $D = (\mathbf{y}\mathbf{y}^\top) \odot (XX^\top)$ positive definite? Either prove this statement or give a counterexample.

2. **(5 pts)** Derive the dual problem for (52)–(54), the case with soft margins.

3. **(5 pts)** Let $(\mathbf{p}^*, \boldsymbol{\lambda}^*)$ be the solution to the modified KKT system

$$\begin{bmatrix} \tilde{H} & A^\top \\ A & 0 \end{bmatrix} \begin{bmatrix} -\mathbf{p} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \nabla f \\ 0 \end{bmatrix}.$$

(see the end of Section 4). Show that $\mathbf{p}^*$ is a descend direction i.e., $\nabla f^\top \mathbf{p} < 0$ (meaning that motion along it for a sufficiently short distance will reduce the value of the objective function) provided that columns of $A^\top$ are linearly independent and $n < d$. *Hint: First try to get it yourself. If you get stuck, look into M. Benzi, G. H. Golub, and J. Liesen, "Numerical solution of saddle point problems", Acta Numerica, pp. 1–137, 2005.*

4. **(5 pts)** Consider a Swiss Roll dataset shown Fig. 7(a). This dataset is generated by the provided Matlab code `stardata.m`. Design a nonlinear mapping to 2-dimensional or 3-dimensional feature space in which the blue and black sets are separable by a line or a plane. Visualize the data in the feature space so that it is apparent that there exists a line or a plane separating them. Submit a formula for your nonlinear map and your figure with the data mapped to the feature space. *You can also draw a linear divider but I am not requiring this at this time. I am also providing a text file with data set whose rows are $x_i(1), x_i(2), y_i$, $i = 1, \ldots, 600$, for the case you need it.*