Homework 3. Due Thursday, Oct. 15

1. **(5 pts)** Consider the linear least squares problem

$$\min_{\mathbf{x} \in \mathbb{R}^d} \frac{1}{2} \|A\mathbf{x} - \mathbf{b}\|_2^2 \tag{1}$$

   where A is $n \times d$ and has rank $k \leq \min\{n, d\}$. Prove that the solution to (1) is given by

$$\mathbf{x}^* = V_k \Sigma_k^{-1} U_k^\top \mathbf{b}$$

   where U_k and V_k are the matrices consisting of the first k columns of U and V and Σ_k is the top-left $k \times k$ submatrix of Σ in an SVD of A.

2. **(5 pts)** Consider the constrained minimization problem

$$\min_{\mathbf{x} \in \mathbb{R}^d} \frac{1}{2} \|A\mathbf{x} - \mathbf{b}\|_2^2 \quad \text{subject to} \quad t - \|\mathbf{x}\|_1 \geq 0, \tag{2}$$

   where t is a given constant. Setup a quadratic programming problem to solve (2). Do it using the decomposition: $\mathbf{x} = \mathbf{x}_+ - \mathbf{x}_-$ where $\mathbf{x}_+$ and $\mathbf{x}_-$ are vectors with nonnegative components. You do not need to solve it.

3. **(5 pts)** Reproduce any example from Lagaris, Likas, and Fotiadis, Section IV, of your choice except for Problem 5 that I reproduced. The choice of the architecture of the NN, optimization method, and software is up to you. All examples in this paper are solved using NNs with only one hidden layer. My Matlab codes for Problem 5 are posted on ELMS in Files/Codes/NLLS. In your report, write the setup for the optimization problem and specify optimization method and software developed or used, and compare your solution with the exact solution and with the numerical solution in this paper.