Dipole excitations of hydrodynamic electrons in graphene

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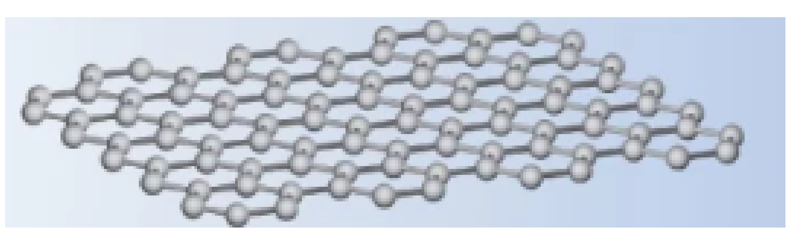
Electron hydrodynamics in graphene

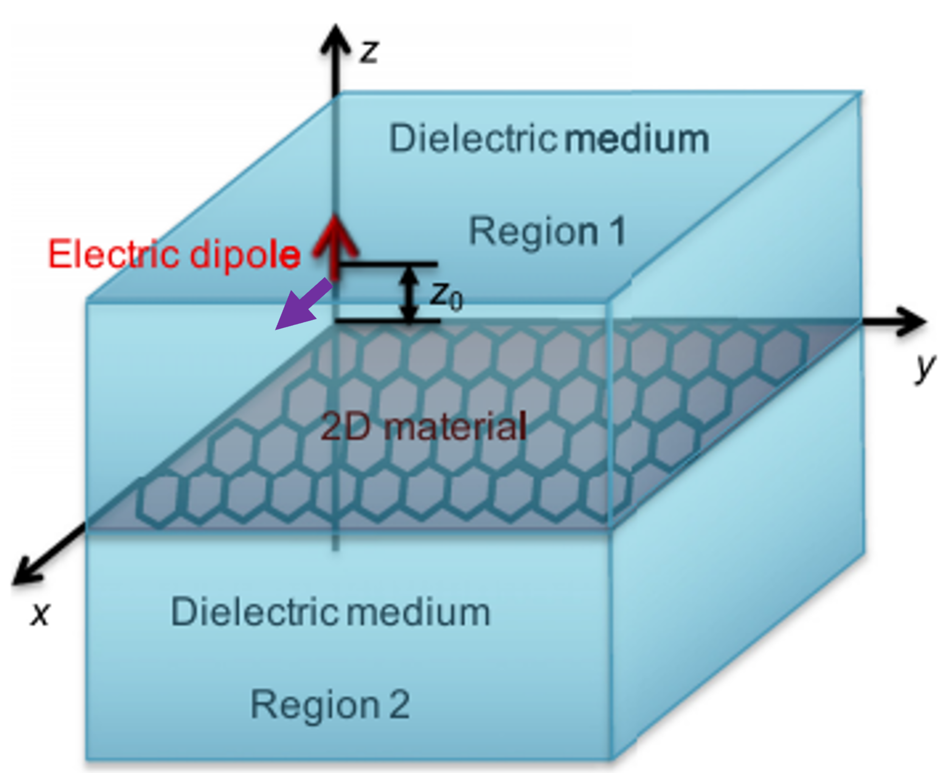
- In **conductors**, resistance arises because electrons collide with the atomic nuclei of the crystal lattice.
- **Graphene** is a two-dimensional, stable allotrope of carbon that has novel applications to the development of **nanodevices** (see Fig 1).
- Notably, it has been experimentally observed that in **graphene**, internal collisions between electrons dominate and the electron system may behave like a **fluid** [1].
- **Objective**: Develop **analytical methods** that can be used to help experimentalists **detect** and **control** electron fluid behavior [2].

Figure 1. Structure of graphene. Graphene consists of a planar network of carbon nuclei on which electrons can flow [Geim, Grigorieva, *Nature*, 2013].



Dipole Excitations: Geometry

Figure 2. Schematic of geometry. A vertical or horizontal **electric dipole** (indicated by thick arrows) is placed at distance z_0 above an infinite graphene sheet [2].



Project Goal

Explicitly solve Maxwell's equations and Navier-Stokes equations for the electron fluid to obtain the electromagnetic fields and hydrodynamic modes produced by a horizontal dipole on a graphene sheet (see Fig 2).

Projected Maxwell-Navier-Stokes Equations

Maxwell's Equations in 3 dimensions (**outside** graphene):

$$\begin{cases} \nabla \times \mathbf{H}_j = -\frac{i\omega\varepsilon}{c}\mathbf{E}_j + \frac{4\pi}{c}\mathbf{J} \\ \nabla \times \mathbf{E}_j = \frac{i\omega}{c}\mathbf{H}_j \end{cases} \qquad \mathbf{J} = \begin{cases} \delta(x)\delta(y)\delta(z-z_0)\mathbf{e}_z(\text{vertical}) \\ \delta(x)\delta(y)\delta(z-z_0)\mathbf{e}_x(\text{horizontal}) \end{cases}$$

Linearized **Navier-Stokes Equations** for electron fluid in 2 dimensions (**on** graphene):

$$\begin{cases} -i\omega n + n_0\nabla\cdot\mathbf{v} = 0 \\ -i\omega\mathbf{v} = -\frac{s^2\nabla n}{n_0} - (\gamma - \eta\nabla^2)\mathbf{v} + \zeta\nabla(\nabla\cdot\mathbf{v}) - (\omega_c - \eta_H\nabla^2)\mathbf{v}\times\mathbf{e}_z - \frac{e\mathbf{E}_\parallel}{m} \end{cases}$$

Effective Boundary Condition for Maxwell's Equations (**on** graphene):

$$\begin{cases} (\mathbf{E}_2 - \mathbf{E}_1)\times\mathbf{e}_z = \mathbf{0} \\ (\mathbf{H}_2 - \mathbf{H}_1)\times\mathbf{e}_z = \frac{4\pi}{c}\mathbf{j}^s \end{cases}$$

Surface current density:

$$\mathbf{j}^s = -en_0\mathbf{v}$$

Methods

- We approach solving this PDE system by distinguishing two parts of the solution:
  - The scattered fields (or **homogeneous solutions**) are the general solutions to the system in the **absence of the dipole** (but in the presence of the material).
  - The primary fields (or **particular solutions**) are particular solutions to the system in the **absence of the material sheet** (but in the presence of the dipole).
- To aid with solving the PDEs, we use the **Fourier Transform** in (x, y):

$$\mathcal{F}\{f(\mathbf{r},z)\} = \hat{f}(\mathbf{k},z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathbf{r},z)e^{-i\mathbf{k}\cdot\mathbf{r}}dxdy$$

- The Fourier Transform converts PDEs to **algebraic systems** and **ODEs**.
- **Key Result:** Taking the Fourier transform of the Navier-Stokes equations gives the following Ohm's Law-type formula:

$$\hat{\mathbf{j}}^s(\mathbf{k}) = \hat{\underline{\sigma}}(\mathbf{k};\omega)\hat{\mathbf{E}}_\parallel(\mathbf{k})$$

  where $\hat{\underline{\sigma}}(\mathbf{k};\omega)$ is a **conductivity tensor**.
- To solve for the fields explicitly, we **invert the Fourier Transform**:

$$\mathcal{F}^{-1}\{\hat{f}(\mathbf{k},z)\} = f(\mathbf{r},z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{f}(\mathbf{k},z)e^{i\mathbf{k}\cdot\mathbf{r}}dk_xdk_y$$

  Evaluating such integrals requires tools from **complex analysis**, including **analytic function** theory.

Results in Fourier-Bessel Representation

Vertical Dipole

$$E_{1z}(r,z) = \frac{i}{\omega\varepsilon}\int_0^\infty \frac{k^3}{\beta}J_1(kr)\left[\frac{\mathcal{A}(k)+\mathcal{D}(k)}{\mathcal{D}(k)}e^{-\beta(z+z_0)} + e^{-\beta|z-z_0|}\right]dk$$

$$E_{2z}(r,z) = \frac{i}{\omega\varepsilon}\int_0^\infty k^3J_1(kr)\frac{\mathcal{A}(k)}{\beta\mathcal{D}(k)}e^{\beta(z-z_0)}dk$$

Horizontal Dipole

$$E_{1z}(r,\phi,z) = \frac{i}{\omega\varepsilon}\int_0^\infty k^2J_1(kr)\frac{\mathcal{M}(k)\cos\phi - \mathcal{E}(k)\sin\phi}{\mathcal{D}(k)}e^{-\beta(z+z_0)}dk$$
$$+\frac{i}{\omega\varepsilon}\int_0^\infty k^2J_1(kr)\mathrm{sgn}(z-z_0)\cos(\phi)e^{-\beta|z-z_0|}dk$$

$$E_{2z}(r,\phi,z) = \frac{i}{\omega\varepsilon}\int_0^\infty k^2J_1(kr)\frac{\mathcal{C}(k)\cos\phi + \mathcal{E}(k)\sin\phi}{\mathcal{D}(k)}e^{\beta(z-z_0)}dk$$

The **hydrodynamic modes** are given by the **poles** of the integrand:

Diffusive Mode

$$k_d(\omega) \simeq \sqrt{\frac{i\omega-\gamma}{\eta} + \frac{\omega^2}{c^2}(\epsilon-1)}$$

Plasmon mode

$$k_{pl}(\omega) \simeq \frac{\omega\varepsilon(\omega+i\gamma)}{D_0}\left(1 - i\eta\frac{\omega^2(\omega+i\gamma)\varepsilon^2}{D_0^2}\right)$$

Conclusions

- The hydrodynamic modes are the **same** in both the vertical and horizontal cases.
- However, preliminary examinations suggest that the strengths of the modes are **weaker** for the horizontal dipole, though further numerics are required to verify this.

References

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