AMSC/MATH 673, CLASSICAL METHODS IN PDE, FALL 2018.

MWF 2:00pm - 2:50pm
MTH 0407
Instructor: Matei Machedon
Office: MTH 3311
Office hours: Fridays 11-12 and by appointment.
e-mail: mxm@math.umd.edu

Required text: Evans, Partial Differential Equations **second edition**
Recommended: Hörmander, The analysis of linear partial differential operators, volume I (any edition). (See also the notes on distribution theory, available on my web page)

This was a "qualifying exam" course on theoretical methods in PDEs. Most theory will cover linear equations (with the exception of first order nonlinear equations), but some homework problems will illustrate applications to non-linear problems.

The prerequisites for this course are a working knowledge of multivariable calculus, and mathematical maturity. Some familiarity with undergraduate PDEs, real analysis and complex variables (harmonic functions in particular) would also be helpful.

The catalog description of this course is
"Analysis of boundary value problems for Laplace's equation, initial value problems for the heat and wave equations. Fundamental solutions, maximum principles, energy methods. First order nonlinear PDE, conservation laws. Characteristics, shock formation, weak solutions. Distributions, Fourier transform", and we will come close to covering exactly that.

More specifically, will cover the following topics:

"Classical" results, from Evans' book:
Fundamental solution of Laplace equations in the whole space.
Mean value theorem and maximum principle for the Laplace equation.
Green's functions for the Laplace equation.

Energy methods for the Laplace equation.
Fundamental solution for the heat equation in the whole space.
Classical maximum principle for the heat equation.
Energy methods for the heat equation.
Fundamental solution for the 1d wave equation in the whole space
Fundamental solution for the 2d, 3d wave equation in the whole space, possibly without proofs.
Energy method for the multi-d wave equation
The general method of characteristics
Characteristics for transport equations in the whole space with non constant coefficients.
Weak solutions for transport equations
Maximum principle for classical solutions to 2nd order linear elliptic equations with smooth coefficients
The above topics will be taken from the following chapters in Evans:
Chapter 2: all sections, except some topics in 2.3.2, 2.3.3, and the fundamental solution of the wave equation in high dimensions.
Chapter 3: 3.2, some 3.3 and 3.4
Chapter 6: 6.4
as well as background from Appendix A, B, C.

The following topics will follow Hörmander's book (or my notes)
Notions of weak solutions, distributions, test functions, Schwartz functions.
Weak solutions to Laplace, wave and heat equations, using the fundamental solution.
Fourier transform on L^2, Schwartz class
Solution to the multi-d wave equation in L^2 by Fourier transform.

Additional topics may be covered, as time permits.

There will be several problem sets, two in-class exams and an in-class final. Some problems will be taken from old qualifying exams, available at
http://www-math.umd.edu/quals.html

Grading: 20 % homework, 40% mid-term exams, 40% final exam.

In-class exams: on Monday, October 22 and Monday December 3.
The final exam will be on Saturday, December 15
1:30-3:30pm

Students who require special examination conditions must register with the office of Accessibility and Disability Services (ADS) in Shoemaker Hall. Documentation must be provided to the instructor. Proper forms must be filled and provided to the instructor before every exam.

The Universitys policy on religious observance and classroom and tests states that students should not be penalized for participation in religious observances. Students are responsible for notifying the instructor of projected absences within the first two weeks of the semester. This is especially important for final examinations.

Homework assignments:

Problem set 1 (Laplace's equation), due Friday September 21.
Evans:
2.5: 3, 4, 5, 6, 7
Qualifying exams:
January 2010, Problem 1
January 2005, Problem 1
August 2004, Problem 1

Problem set 2, due Friday, October 5 (heat equation)
2.5: 13, 14, (we may discuss 15 in class, but this is not to be turned in)
Qualifying exams:
January 13, Problem 4 (you may assume $c > 0$)
January 12, Problem 2 (There is a typo. The RHS should read $\frac{1}{\sqrt{2t+1/c^2}}$).
August 11, Problem 3
August 05 Problem 6

Problem set 3, due Friday, October 12 (wave equation)
2.5: 24
Also, if u is smooth and $\Box u = 0$ in $\mathbb{R}^3 \times (0, \infty)$, $u(0, .) = 0, u_t(0, .) = f \in C_0^\infty$, prove there exists C, independent of f , such that

$$|u(t, x)| \leq \frac{C}{t} \int_{\mathbb{R}^3} |\nabla f|$$

for all $t > 0, x \in \mathbb{R}^3$.
Qualifying exams:
January 07, Problem 2
August 03, Problems 4 and 5.
August 06 Problem 3

August 01 Problem 1

Exam 1 Monday October 22
Homework for Chapter 3.
You are responsible for:
All 3.2, 3.4.1, 3.4.3 up to the statement (but not the proof) of Theorem 3 (p. 151), 3.4.4.
I will discuss some 3.3, but this "won't be on the exam". Also, you may want to read the statements of the rest of the theorems, but skip the proofs.
Problem set 4, due Wednesday. November 14: Qualifying exams
Aug 06, problem 1
Jan 05, problem 2b
Jan 2000, problem 2
Aug 98 problem 5
Problem set 5, due Wednesday, November 28.
Qualifying exams:
Aug 96, 1a
Evans, Ch 6: 9, 10 (= 6, 7 in the first edition)
To be solved in class : January 2000, problem 1.
Exam 2 Monday December 3
The final exam will be on Saturday, December 15
1:30-3:30pm