AMSC/MATH 673, CLASSICAL METHODS IN PDE, FALL 2018.

MWF 2:00pm - 2:50pm
MTH 0407
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Required text: Evans, Partial Differential Equations second edition
Recommended: Hörmander, The analysis of linear partial differential operators, volume I (any edition). (See also the notes on distribution theory, available on my web page)

This was a "qualifying exam" course on theoretical methods in PDEs. Most theory will cover linear equations (with the exception of first order nonlinear equations), but some homework problems will illustrate applications to non-linear problems.

The prerequisites for this course are a working knowledge of multivariable calculus, and mathematical maturity. Some familiarity with undergraduate PDEs, real analysis and complex variables (harmonic functions in particular) would also be helpful.

We will cover the following topics.
"Classical" results, from Evans’ book:
Fundamental solution of Laplace equations in the whole space.
Mean value theorem and maximum principle for the Laplace equation.
Green’s functions for the Laplace equation.
Energy methods for the Laplace equation.
Fundamental solution for the heat equation in the whole space.
Classical maximum principle for the heat equation.
Energy methods for the heat equation.
Fundamental solution for the 1d wave equation in the whole space
Fundamental solution for the 2d, 3d wave equation in the whole space, possibly without proofs.
Energy method for the multi-d wave equation
The general method of characteristics
Characteristics for transport equations in the whole space with non constant coefficients.

Weak solutions for transport equations

Maximum principle for classical solutions to 2nd order linear elliptic equations with smooth coefficients

The above topics will be taken from the following chapters in Evans:
- Chapter 2: all sections, except some topics in 2.3.2, 2.3.3, and the fundamental solution of the wave equation in high dimensions.
- Chapter 3: 3.2, some 3.3 and 3.4
- Chapter 6: 6.4
as well as background from Appendix A, B, C.

The following topics will follow Hörmander’s book (or my notes)
- Notions of weak solutions, distributions, test functions, Schwartz functions.
- Weak solutions to Laplace, wave and heat equations, using the fundamental solution.
- Fourier transform on $L^2$, Schwartz class
- Solution to the multi-d wave equation in $L^2$ by Fourier transform.

Additional topics may be covered, as time permits.

There will be several problem sets, two in-class exams and an in-class final. Some problems will be taken from old qualifying exams, available at

http://www-math.umd.edu/quals.html