## AMSC/MATH 674 0101, SPRING 2018

MWF 12-12:50 MTH 1313
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Office hours Mondays and Wednesdays 1-2

Required text: Evans, *Partial Differential Equations* **second edition**
Strongly recommended: Hörmander, *The analysis of linear partial differential operators*, volume I (any edition). I will provide notes which serve as a guide to the topics in distribution theory we will cover. The notes are posted on my web page.
Also recommended: Zimmer, *Essential results of functional analysis*, and Wheeden and Zygmund, *Measure and Integral* (real analysis background).

This used to be a "qualifying exam" course on theoretical methods in PDEs, emphasizing methods based on functional analysis and Soboleb space. The prerequisites for this course are math 673 (or equivalent), but familiarity with real variables (Math 630) and mathematical maturity are sufficient. Some familiarity with functional analysis would be very helpful, but we will cover all necessary topics.

This year[1] will cover the following topics.
$W^{1,p}$ Sobolev spaces in a domain: Definition, extensions, traces, embedding, Poincare's inequality.
$H^s$ Sobolev spaces using the Fourier transform.
Lax-Milgram theorem. Variational solutions to elliptic equations (solving a PDE using the Riesz Representation Theorem in a Hilbert space).
Elements of spectral theory for bounded self-adjoint operators: Definition of the spectrum, the case of compact operator (diagonalization, Fredholm alternative).
There will be several problem sets (some taken from old old qualifying exams, available at http://www-math.umd.edu/quals.html), and 2 in-class exams covering these standard topics.

[1]It is likely Math 674 will be re-designed in future years

If time permits, we will cover harmonic analysis/PDE topics such as the Hardy-Littlewood-Sobolev inequality, introduction to Calderon-Zygmund theory for elliptic equations, introduction to Strichartz estimates for the Schrödinger equation, Littlewood-Paley decompositions, connections with complex analysis (the $\bar{\partial}$ equation).

The final exam will be take-home.

Grading: 30 % homework, 40% two in-class exams (on chapters 5 and 6 in Evans), 30% final exam.

Dates for the in-class exams: Monday March 12 and Monday, April 16.

Make-up policy: There will be no make-ups for the in-class exams. In the case of an absence due to illness, religious observance, participation in a University activity at the request of University authorities, or other compelling circumstances, your blank grade will be replaced by the average of your homework grade and the final exam grade, weighted equally.

No late homework will be accepted. Homework assignments missed due to valid reasons will be replaced by the average of the other homework grades.

The major grading events for this class are the in-class exams and the final.

On exams students must write by hand and sign the following pledge:

I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Students who require special examination conditions must register with the office of the Disabled Students Services (DSS) in Shoemaker Hall. Documentation must be provided to the instructor. Proper forms must be filled and provided to the instructor before every exam.

The Universitys policy on religious observance and classroom and tests states that students should not be penalized for participation in religious observances. Students are responsible for notifying the instructor of projected absences within the first two weeks of the semester. This is especially important for final examinations.

I will communicate with the class by e-mail. You can update your e-mail address at http://www.testudo.umd.edu/apps/saddr/

Problem set 1, due in late February, at a date to be determined. (This is the material covered by the first in-class exam)

Evans, Chapter 5: 1, 4, 5, 6, 8, 9, 12, 14 , 15, 16, 20, 21

Qualifying exams:
August 2009, 6
Jan 2009, 6
August 2008, 5
August 2006, 4

Problem set 2, due in early April, at a date to be determined. (More problems may be added. )
Qualifying exams:
August 13, 3, 4
January 2011, 5
January 2007, 6
January 2006, 4
Aug 2003, 6
Aug 2002, 3, 4
Jan 2001, 4
Jan 2000: 5, 6
Also assigned are the following problems:

1 Let $H_1, H_2$ be Hilbert spaces, and $F : H_1 \to H_2$ a bounded linear functional. Prove that if $x_n \in H_1$ converges weakly to $x \in H_1$, then $F(x_n)$ converges weakly to $F(x)$.

2 Let $B$ be the unit ball in $\mathbb{R}^2$, and $S$ its boundary. Prove there exists a constant $C$ such that, for all nice u,

$$\int_B u^2 dxdy \leq C\left( \int_B |\nabla u|^2 dxdy + \int_S u^2 ds \right)$$

3 Find the weak formulation of the boundary value problem $-\Delta u = f$ in $B$, $u + u_n = 0$ on $S$, and prove that for every $f \in L^2$, this problem has a unique $H^1$ solution. Here $u_n = u_r$ is the normal derivative to the boundary.

4 Consider the related problem $-\Delta u = f$ in $B$, $u - u_n = 0$ on $S$. Show by an example, that this problem does not, in general have a unique solution.
TO BE CONTINUED