1. Consider the following three matrices:

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ -2 & 0 & 6 & -2 \\ 4 & -1 & 2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} -9 & 3 \\ 3 & -1 \\ 6 & -2 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix}$$

   (a) For each of A, B and C, determine if the columns are linearly independent or not. Justify each.

   (b) For each of B and C, determine if the columns span $\mathbb{R}^3$. Justify each.

2. Let A be the matrix given here:

$$\begin{bmatrix} -1 & 2 & -2 \\ 1 & -2 & 1 \\ 2 & -4 & 3 \end{bmatrix}$$

   Let $T(\bar{x})$ be defined by $T(\bar{x}) = A\bar{x}$.

   (a) Find $T(-1, 2, 1)$.

   (b) Find all $\bar{x}$ such that $T(\bar{x}) = (5, 4, -1)$. Express your answer in parametric form.

   (c) Find all h so that $(h, h, 1)$ is in the range of T.

3. (a) Suppose T is a linear transformation with $T(x_1, x_2, x_3) = (2x_1 - x_2, -x_1 + x_2 - 3x_3)$. Justify whether or not T is one-to-one.

   (b) Suppose a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by first rotating $\frac{\pi}{6}$ radians clockwise about the origin and then reflecting in the x_2-axis. Find each of the following:

      i. The matrix corresponding to T.

      ii. $T(x_1, x_2)$

4. (a) Suppose A is a 5×3 matrix for a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^5$. Explain why T cannot be onto $\mathbb{R}^5$.

   (b) Suppose $\bar{x}_1$ and $\bar{x}_2$ are both solutions to the matrix equation $A\bar{x} = \bar{b}$. Find two solutions to $A\bar{x} = \bar{0}$

   (c) Show that the transformation $T(x_1, x_2) = (x_1 + 2x_2 + 1, x_2)$ is not linear.

5. (a) Show that $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$.

   (b) Determine which of the following matrices are invertible. Justify.

$$A = \begin{bmatrix} -2 & -4 & 3 \\ 1 & 7 & 75 \\ -2 & -4 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 \\ -2 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

   (c) Use the inverse of a matrix to solve the following system of equations.

$$\begin{aligned} 4x_1 + x_2 &= \alpha \\ -2x_1 + 2x_2 &= \alpha \end{aligned}$$