Math 240 Exam 2 Sample 2 **Spring 2008**

Please put problem 1 on answer sheet 1

1. (a) Find the determinant of

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

using row-reduction. Simplify.

(b) Suppose A and B are both $n \times n$ matrices. Suppose B is invertible and $\mathrm{Det}(A^4 B) = 0$. Explain why A is not invertible.

Please put problem 2 on answer sheet 2

2. (a) Use Cramer's Rule to find x_2 for the system of equations given below.

$$\begin{aligned} 2x_2 + 5x_3 &= 7 \\ -3x_1 + x_2 - x_3 &= 10 \\ x_2 + x_3 &= 2 \end{aligned}$$

(b) Let $T : \mathbb{P}_2 \to \mathbb{R}^2$ be the transformation given by

$$T(p(t)) = \begin{bmatrix} p(0) + p'(0) \\ p'(1/2) \end{bmatrix}$$

i. Show that $T(cp(t)) = cT(p(t))$.

ii. Find a single polynomial $r(t)$ such that Ker T=span$\{r(t)\}$.

Please put problem 3 on answer sheet 3

3. (a) Two of the following statements are false. Identify which are false and justify.

i. $\mathbb{P}_2$ is a subspace of $\mathbb{P}_3$.

ii. $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

iii. The set of vectors of the form $\begin{bmatrix} a \\ b \\ a + 2b + 1 \end{bmatrix}$ with a and b reals numbers is a subspace of $\mathbb{R}^3$.

(b) In the vector space of continuous real-valued functions, justify why the set

$$\left\{3 + \sin^2 x, 1, 5 - 4\cos^2 x\right\}$$

is a linearly dependent set.

Turn Over!

Please put problem 4 on answer sheet 4

4. (a) Define

$$V = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} \right\}$$

   i. Find a matrix A so $V = \text{Col } A$.

   ii. Find a matrix B so $V = \text{Row } B$.

   iii. Find a matrix C so $V = \text{Nul } C$.

   (b) i. Suppose you have a homogeneous system of fifteen equations in twenty variables. What is the smallest that the dimension of the solution set could be? Justify.

   ii. Suppose A is a 3×8 matrix. Why can the null space not have dimension 4?

Please put problem 5 on answer sheet 5

(a) Suppose $\mathcal{B} = \{t^2 + 3t + 1, t + 1, -1\}$ is a basis for $\mathbb{P}_2$. Find $\left[3t^2 + t - 1\right]_{\mathcal{B}}$.

(b) Suppose $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix} \right\}$ and $\mathcal{D} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ are both bases for $\mathbb{R}^2$.

Find $\underset{\mathcal{D} \leftarrow \mathcal{C}}{P}$ and then $\underset{\mathcal{C} \leftarrow \mathcal{D}}{P}$.