Math 240 Exam 3 Sample 2 **Spring 2008**

Please put problem 1 on answer sheet 1

1. (a) Show that if $\{\bar{u}, \bar{v}\}$ is an *orthonormal* set, then $\{2\bar{u}+\bar{v}, 3\bar{u}-6\bar{v}\}$ is an *orthogonal* but not *orthonormal* set.

   (b) Show that $\begin{bmatrix} a & -b \\ b & -a \end{bmatrix}$ has real eigenvalues if and only if $a \geq b$.

Please put problem 2 on answer sheet 2

2. (a) Give an example of each of the following. No explanation is required.

   i. A 2×2 matrix with no eigenvalues.

   ii. A 2×2 matrix with a single eigenvalue for which the eigenspace is 2-dim.

   (b) Use the Gram-Schmidt process to find an orthogonal basis for

$$W = \text{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

   You do not need to simplify your third vector.

Please put problem 3 on answer sheet 3

3. Define

$$A = \begin{bmatrix} 3 & 0 & -2 \\ -1 & 2 & 2 \\ 2 & 0 & -2 \end{bmatrix}$$

   (a) Find the characteristic polynomial for A.

   (b) Suppose that $\lambda = -1$ and $\lambda = 2$ are the eigenvalues for A. Find a basis for each eigenspace. Explain how you know that A is diagonalizable.

   (c) Diagonalize A two different ways.

Please put problem 4 on answer sheet 4

4. (a) Suppose A and B are matrices with B invertible. Furthermore suppose A and B have an eigenvector $\bar{v}$ in common but with different eigenvalues λ_A and λ_B. Find an eigenvalue for AB^{-1}.

   (b) Suppose

$$\bar{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad W = \text{Span}\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

   Write $\bar{u} = \hat{u} + \bar{z}$ with $\hat{u}$ in W and $\bar{z}$ in $W^{\perp}$.

   (c) Find the equation of the least-squares line for the three points $(1, 3)$, $(2, 4)$, $(3, 5)$.

Please put problem 5 on answer sheet 5

5. Define the quadratic form $Q(\bar{x}) = 2\sqrt{2}x_1x_2 + x_2^2$.

   (a) Find the matrix for Q.

   (b) Classify $Q(\bar{x})$ as positive definite, negative definite or indefinite and justify.

   (c) If you were to use a change of variables to convert Q into another quadratic form with no cross-product term, what would the matrix for this new form be? Explain in words how you would get the change-of-variables matrix.
