MATH 240 – Spring 2013 – Exam 2

CALCULATORS ARE NOT ALLOWED
TURN OFF ALL ELECTRONIC DEVICES

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***** THERE ARE QUESTIONS ON BOTH SIDES OF THIS PAPER *****

Answer each question on a separate sheet of paper. Use the back side if necessary. On each sheet, put your name, your section leader's name and your section meeting time. No proof is needed for TRUE-FALSE questions; just write clearly. You may assume given matrix expressions are well defined (i.e. the matrix sizes are compatible).

1. (a) Below are a matrix A and the matrix rref(A) produced by MATLAB.

$$A = \begin{pmatrix} 22 & -22 & 44 & 14 & 7 & 182 \\ 4 & -4 & 8 & 5 & 2 & 37 \\ 14 & -14 & 28 & 4 & 3 & 108 \\ 0 & 0 & 0 & 0 & 10 & 20 \\ 15 & -15 & 30 & 10 & 25 & 165 \end{pmatrix} , \qquad \text{rref}(A) = \begin{pmatrix} 1 & -1 & 2 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} .$$

Write down a basis for each of the following (no justification required).

i. (4 points) Row(A), the row space of A.

ii. (4 points) Col(A), the column space of A.

iii. (6 points) Nul(A), the null space of A.

(b) (6 points) For each of the following, answer TRUE or FALSE.

i. $\text{rank}(A+B) \geq \text{rank}(A)$ whenever A and B are 10×21 matrices.

ii. $\text{rank}(AB) \leq \text{rank}(A)$ whenever A and B are 10×10 matrices.

2. (a) (15 points) Define

$$A = \begin{pmatrix} -3 & -2 \\ 1 & 3 \end{pmatrix} \qquad \text{and} \qquad D = \{x \in \mathbb{R}^2 : \sqrt{(x_1 - \pi)^2 + (x_2 - 17)^2} \leq 3\} .$$

Compute the area of the set $E = \{Ax : x \text{ is in } D\}$.

(b) (5 points) Suppose A and B are 5×5 matrices with $\det(A) = 10$ and $\det(B) = 4$. Compute the determunant of the matrix $M = -2\,A^3 B^{-1}$.

3. (a) (14 points) Let $\mathbb{P}^1$ denote the vector space of polynomials of degree at most 1; then $\mathcal{B} = \{3 + t, 5 + 5t\}$ is a basis of $\mathcal{B}$.
Find the coordinates vector $x = [-7 + t]_{\mathcal{B}}$ of the polynomial $-7 + t$.

(b) (6 points) For each of the following, answer TRUE or FALSE.

i. $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

ii. If $\mathcal{C}$ is a set of 14 vectors which span $\mathbb{R}^5$, then $\mathcal{C}$ contains every basis of $\mathbb{R}^5$.

4. (a) (8 points) Suppose that A and B are similar matrices. Prove that their characteristic polynomials are equal.

   (b) (12 points) Let V be the vector space of differentiable functions from $\mathbb{R}$ to $\mathbb{R}$. For each of the following, answer TRUE or FALSE.

      i. The set of functions $\{\cos^2 t, \sin^2 t\}$ is a linearly independent subset of V.

      ii. The map $T : V \to \mathbb{R}$ defined by the rule $T(f) = f'(1)$ is a linear transformation.

      iii. The map $T : V \to \mathbb{R}$ defined by the rule $T(f) = f(1) - 1$ is a linear transformation.

      iv. If S is a nonempty subset of a vector space V, then the set of all linear combnations of S is a subspace of V.

5. (a) (8 points) The determinant of an $n \times n$ matrix A is a polynomial function of the entries of A.

      i. What is the degree of this polynomial, if $n = 5$?

      ii. This polynomial is a sum of monomials; how many monomials are there in this sum, if $n = 5$?

   (b) (12 points) For each of the following, answer TRUE or FALSE.

      i. If A is a 2×2 matrix with no eigenvalue, then $\det(A) > 0$.

      ii. If 3 is an eigenvalue of a 4×4 matrix A, then 15 is an eigenvalue of $5A$.

      iii. If A and B are 2×2 matrices with equal characteristic polynomials, then A and B are similar matrices.

      iv. Two finite dimensional vector spaces are isomorphic vector spaces if they have the same dimension.