

HYPOTHESIS TESTS: EXAMPLES

Problem 26, Ch. 8, Devore.

Here a specific α is not specified. Let us pick $\alpha = .05$

1. Parameter. $\mu =$ population average penetration in mils.
2. $H_0 : \mu = 50$, $H_a : \mu > 50$
3. Test statistic. $Z = (\bar{X} - \mu_0)/(S/\sqrt{n}) = (\bar{X} - 50)/(S/\sqrt{45}) \sim \mathcal{N}(0, 1)$, by CLT
4. Rejection region = $[z_\alpha, \infty) = [z_{.05}, \infty) = [1.64, \infty)$.
5. Compute: $z = (52.7 - 50)/(4.8/\sqrt{45}) = 3.77$.
6. Conclude: Reject H_0 . Don't use these conduits.
7. P-value is approximately .0001 . VERY strong support for rejecting H_0 .

Problem 31, Ch. 8, Devore.

1. Parameter. $\mu =$ population average adjustment time in seconds.
2. $H_0 : \mu = 7$, $H_a : \mu < 7$
3. Test statistic. $T = (\bar{X} - \mu_0)/(S/\sqrt{n}) = (\bar{X} - 7)/(S/\sqrt{9}) \sim t_{\nu=8}$
4. Rejection region = $(-\infty, t_{\alpha, \nu}) = (-\infty, t_{.1, 8}) = (-\infty, -1.397]$.
5. Compute: $t = (6.32 - 7)/(1.65/\sqrt{9}) = -1.24$.
6. Conclude: Retain H_0 . Data does not contradict prior belief at $\alpha = .1$
7. P-value is not much smaller than .1. Does not provide additional strong support for rejecting H_0 .

Problem 37, Ch. 8, Devore.

1. Parameter. $p =$ population proportion of type A among donors
2. $H_0 : p = .40$, $H_a : p \neq .40$
3. Test statistic. $Z = (\hat{p} - p_0)/(\sqrt{p_0(1 - p_0)}/\sqrt{n}) = (\hat{p} - .40)/(\sqrt{.40(.60)}/\sqrt{150}) \sim \mathcal{N}(0, 1)$, by CLT
4. Rejection region = $\{z : |z| \geq z_{\alpha/2}\} = \{z : |z| \geq z_{.005}\} = \{z : |z| \geq 2.58\} = (-\infty, -2.58] \cup [2.58, \infty)$
5. Compute: $z = ((82/150) - .40)/(\sqrt{.40(.60)}/\sqrt{150}) = 3.67$.
6. Conclude: Reject H_0 at $\alpha = .01$. Conclude actual percentage of Type A donors is not .40 .
7. P-value is less than .0002 . Very strong support for conclusion.