This regards the lecture series we are invited to give during the G2D2-2019 summer school and conference “Groups and Graphs, Designs and Dynamics”, to take place at Three Gorges University, Yichang City, China, August 12–25, 2019.

We plan to give an exposition of the relationships between algebraic invariants of matrices and dynamical invariants of the symbolic dynamical systems they define. We will focus in particular on shift equivalence, and strong shift equivalence, from several viewpoints, including polynomial matrices and algebraic K-theory. We will design the lectures such that the algebraic discussion can be of general interest, beyond application to symbolic dynamics. We will also discuss inverse problems for primitive nonnegative matrices, including the inverse spectral problem and generalizations, for matrices over the reals and other rings.

The lectures will be too ambitious for complete proofs. We’ll focus on communicating the ideas, invariants and statements. But we will give references (or provide appendices) with the proofs. We hope the lectures might be an entry to problems for some, and a reliable entry reference for all.

There is a preliminary outline of the lectures on the next page. The final content might change somewhat.
Symbolic dynamics and the stable algebra of matrices

I. Basics.
Edge shift of finite type (SFT) $S_A$ defined by square matrix $A$ over $\mathbb{Z}^+$. Correspondence of matrix invariants of $A$ to dynamical invariants of $S_A$. Periodic points, zeta function, $\det(I-zA)$. Strong shift equivalence and shift equivalence. The Williams Conjecture. The Classification Problem for mixing SFTs and its status. Amalgamation, subdivision and strong shift equivalence.

II. Shift equivalence and direct limit modules.

III. Other rings for other symbolic dynamical systems.
Examples. Matrices over group rings present group extensions. Other symbolic dynamical systems. The matrix invariants.

IV. Polynomial matrix presentations and invariants.
Positive K-theory. How the invariants present in this polynomial world. The functor from shift equivalence to flow equivalence.

V. A brief introduction to algebraic K-theory.
This is geared to prepare for our application.

VI. The algebraic K-theoretic characterization of the refinement of strong shift equivalence over a ring by shift equivalence.
With applications to symbolic dynamical systems.

VII. Inverse problems for nonnegative matrices.

VIII. Wagoner’s strong shift equivalence complex, and applications.
Automorphisms of the shift, the dimension representation and SGCC. The counterexamples to Williams conjecture. Connection to algebraic K-theory.