Spring 2012 - Math 437 Section 0101

Homework #3 - **Due Tuesday February 21st**

1. Let $F(x, y, z) = (x + y + z - 1)^2$. Locate the critical points of F. For what values of c is the set $F^{-1}(c)$ a regular surface?

2. For each of the following, determine whether S is a regular surface or not. Justify your answer carefully

   (a) $\{(x, y, z) \in \mathbb{R}^3\,;\, x^3 + y^3 + z^3 - 3xyz = 1\}$

   (b) $\{(x, y, z) \in \mathbb{R}^3\,;\, x = 0 \text{ or } y = 0\}$

3. Show that the Torus (seen in class) is a surface of revolution, and use this fact to give a parametrization. Clearly identify the neighborhood covered by your parametrization and determine how many such parametrization you would need to cover the whole torus.

4. Let $U = \{(x, y, z) \in \mathbb{R}^3\,;\, x > 0,\ y > 0,\ z > 0\}$ and let g and h be defined on U by $g(x, y, z) = x^2 + y^2 + z^2 - 1$, $h(x, y, z) = x - y - z$. Show that the set of all $(x, y, z) \in U$ such that $g(x, y, z) = h(x, y, z) = 0$ is a manifold in $\mathbb{R}^3$. What is the dimension of this manifold.

5. Show that the paraboloid $z = x^2 + y^2$ is diffeomorphic to a plane.

6. Determine the tangent planes of the surface defined by $x^2 + y^2 - z^2 = 1$ at the points $(x, y, 0)$ and show that they are all parallel to the z axis.

7. Let $\mathbb{S}^2 = \{(x, y, z) \in \mathbb{R}^3\,;\, x^2 + y^2 + z^2 = 1\}$ be the unit sphere and let $A : \mathbb{S}^2 \to \mathbb{S}^2$ be the (antipodal) map $A(x, y, z) = (-x, -y, -z)$. Prove that A is a diffeomorphism and compute its derivative.

8. Let S be a regular surface in $\mathbb{R}^3$ and let $f : S \to \mathbb{R}$ be given by $f(p) = |p - p_0|^2$, where $p \in S$ and p_0 is a fixed point of $\mathbb{R}^3$. Show that f is differentiable and that $df_p(w) = 2w \cdot (p - p_0)$ for all $w \in T_pS$.