Assignment #1, due Wednesday, Sept. 14

1. Assume that the effective annual interest rate is 5%. For the next 15 years you will pay monthly 400$ into this account (now, after 1 month,..., after 179 months). You then want to retire. What amount A can you then monthly withdraw from your account for the next 20 years (after 180 months,..., after 419 months)?

2.

   (a) A bank gives you a loan of 1000$ now. You have to make a payment of 500$ after 6 months, and a payment of 600$ after 12 months. What is the effective annual interest rate? (Solve a quadratic equation for β).

   (b) A bank gives you a loan of 1000$ now. You have to make a payment of 500$ after 9 months, and a payment of 600$ after 12 months. What is the effective annual interest rate? (Set up an equation for β and solve it with `fzero` in Matlab).

   (c) A bank gives you a loan of 1000$ now. You have to make payments of 100$ after 4 months, 5 months, ...,8 months. Then you have to make payments of 150$ after 9 months, 10 months, 11 months, 12 months. What is the effective annual interest rate? (Set up an equation for β and solve it with `fzero` in Matlab).

3. We have a "biased coin" which shows "heads" 40% and "tails" 60% of the time. We toss this coin four times. Let Z_j be equal to 1 if the jth toss is "heads", and 0 if it is "tails". Note that Z_1, Z_2, Z_3, Z_4 are independent. Let $X_0 = 0$ and $X_j = Z_1 + \cdots + Z_j$ for $j = 1, \ldots, 4$.

   (a) Find $P(X_4 = x)$ for all possible values x. Use this to find $E[X_4]$ and $Var[X_4]$.

   (b) Find $E[Z_1]$ and $Var[Z_1]$. Then write $E[X_4]$ in terms of $E[Z_j]$, and write $Var[X_4]$ in terms of $Var[Z_j]$. If we toss the coin m times, what would be $E[X_m]$ and $Var[X_m]$?

   (c) Find the conditional expectation $E[X_4 \mid X_1 = 1]$.