Assignment #2, due Wed, Oct. 12

1. Consider stock prices S_0, S_1, S_2, S_3 described by a binomial tree with $S_0 = 8$, $u = \frac{3}{2}$, $d = \frac{1}{2}$. Let $\rho = 20\%$. We consider options with maturity $T = t_N$ with $N = 3$.

(a) Find the price of a "**lookback put option**" with payoff $V_3 = (7 - \min\{S_0, \ldots, S_3\})^+$. *Hint:* This payoff is **path dependent**, hence you have to use the full tree with $1, 2, 4, 8$ possibilities at times t_0, t_1, t_2, t_3.

(b) Show that you can **replicate the payoff** V_3 from (a) with an **investment strategy** (x_0, x_1, x_2): At time t_j (after we know the value of S_j) we choose a portfolio with x_j stocks and an amount y_j in the bank account, for $j = 0, 1, 2$. Label each node in the tree with the number x_j (see "Delta-hedging" in the notes). For the case $(S_0, S_1, S_2, S_3) = (8, 4, 2, 1)$ give the values x_j, y_j for $j = 0, 1, 2$ and the value of the portfolio U_j for $j = 0, 1, 2, 3$.

(c) Find the price of a **European put option** with strike $K = 7$. *Hint:* The payoff in (c), (d) depends only on S_3 (path independent). Use the binomial tree with $1, 2, 3, 4$ possibilities at times t_0, t_1, t_2, t_3 (as shown in the class notes).

(d) Find the price of an **American put option** with strike $K = 7$. Use the binomial tree.

(e) Compare the option prices in (a),(c),(d): Which is smallest, which is largest? Explain the reason for this.

(f) Extend the tree from (d) by using one additional V_3^4 and one additional value V_3^{-1} below. Explain how V_0^1 and V_0^{-1} give option prices for t_0 with $S_0 = 8$, but with different strikes.

(g) Find the price of a **European call option** with strike $K = 7$ for all nodes in the tree from (c) by using the **put-call parity**.

2. Consider stock prices described by a binomial tree with $S_0 = 10$, $u = 1.1$, $d = 0.9$. Let $\rho = 1\%$.We consider options with maturity $T = t_N$ with $N = 12$. Use the m-file `xbinom_eu_call.m` and modify it for the other cases. Pick m so that the smallest $\tilde{S}_0$ is < 4 and the largest $\tilde{S}_0$ is > 25.

(a) Consider a **European put option** with strike $K = 11$: plot the option price vs. initial stock price s, together with function $(1 + \rho)^{-N}K - s$. Also make a plot of option price vs. strike.

(b) Consider a **European call option** with strike $K = 11$: plot the option price vs. initial stock price s, together with function $s - (1 + \rho)^{-N}K$. Also make a plot of option price vs. strike.

(c) Consider a **American put option** with strike $K = 11$: plot the option price vs. initial stock price s, together with the payoff function $H(s)$. Also make a plot of option price vs. strike. Compare with the European put option.

(d) Consider a **American call option** with strike $K = 11$: plot the option price vs. initial stock price s, together with the payoff function $H(s)$. Also make a plot of option price vs. strike. Compare with the European call option.