

# AMSC 424, Fall 2016

## Practice problems for Exam #1

(No calculators allowed for exam)

1.
  - (a) Let  $\rho$  denote the monthly interest rate and  $\beta := \frac{1}{1 + \rho}$ . Give a formula for  $\beta$  in terms of (i)  $r_{\text{eff}}$ , (ii)  $r_c$ . Here  $r_{\text{eff}}$  denotes the yearly effective interest rate, and  $r_c$  denotes the yearly interest rate for continuous compounding.
  - (b) You get a loan of 1000\$ now. You make a payment  $P$  at the end of month 8, month 9, month 10. At the end of month 12 you make a final payment of 200\$. Assume you know  $\beta$  and find the payment  $P$  in terms of  $\beta$ .
2. We use a biased coin which gives “heads” with probability  $\frac{2}{3}$  and “tails” with probability  $\frac{1}{3}$ . We toss the coin twice. You win the amount  $X$  where  $X$  is the number of “tails”.
  - (a) Find  $E[X]$  and  $\text{Var}[X]$ .
  - (b) Let  $A$  denote the event “at least one coin shows heads”. Find the conditional expectation  $E[X | A]$ .
3. At time  $t_0$  the price of a European call option with strike 10 is  $V_0$ , and the price of a European call option with strike 15 is  $\tilde{V}_0$ . Both options have the same maturity  $t_N = N\Delta t$ , the interest rate is  $\rho$  per period of  $\Delta t$ . Prove an inequality  $V_0 \leq \tilde{V}_0 + \dots$  using the comparison principle.  
**Hint:** *Investment 1:* At time  $t_0$  buy a call option with strike 10. *Investment 2:* at time  $t_0$  buy a call option with strike 15 and put a certain amount  $z$  in the bank account.
4. The interest rate is  $\rho = 50\%$  per period  $\Delta t$ . A stock has at time  $t_0$  the price  $S_0 = 4$  and follows a binomial tree model with  $u = 2$  and  $d = \frac{1}{2}$ . We consider options with maturity at  $t_2$  and strike  $K = 4$ .
  - (a) Find the initial price  $V_0^{EP}$  of a European put option. Give the answer as a fraction.
  - (b) Find the price  $V_0^{AP}$  of an American put option. Give the answer as a fraction. Describe the optimal exercise strategy.