ERRATA AND SUGGESTION SHEETS
Advanced Calculus, Second Edition

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- **Page 11, line 38**: "$b^2 < r$" should be "$b^2 < c$"
- **Page 13, line 1**: for "... number a and b," write "... numbers a and b,"
- **Page 16, 1c**: "$\mathbb{Q} \setminus \mathbb{N}$" should read "$\mathbb{Q} \setminus \mathbb{Z}$"
- **Page 18, line 14**: "$1 - T$" should read "$1 - r$."
- **Page 19**: The Geometric sum formula. Left hand side is not define when $r = 0$.
- **Pages 30, 32**: To slightly improve clarity, the Linearity Property should come before Theorem 2.13.
- **Page 32, line 16**: "$p(x) = \sum_{i=0}^{k} c_i x^i$" should appear "$p(x) = c_0 + \sum_{i=1}^{k} c_i x^i$"
- **Page 36, line 16**: "$x = (a + b)/2$" should be "$s = (a + b)/2$".
- **Page 38**: Theorem 2.25 (i) and (ii) should have n in NATURAL numbers $\mathbb{N}$.
- **Page 40, line 2**: "$S_4 + \frac{1}{2} = 1 + \frac{3}{2}$" read "$S_4 + \frac{1}{2} \geq 1 + \frac{3}{2}$."
- **Page 54, line 9**: "sequence $(\{-1/n\}$" should read "sequence $\{-1/n\}$"
- **Page 57, line 14**: "... $f + g : R$ and ..." should read "... $f + g : R \to R$ and ..."
- **Page 62, line 7**: "the value 0." should read "zero or negative values."
- **Page 67, line 21**: "$1/n$" should read "$-1/n$."
- **Page 67, line 22**: "$2 + 1/n^2$" should read "$-2 - 1/n^2$."
- **Page 67, line 24**: "$(0, 1)$" should read "$(0, 2)$."
- **Page 73**: Theorem 3.22 in the first sentence after "ii", it reads "... criterion at the domain D; ..." and it should read "... criterion on the domain D; ..."
- **Page 78, line 22**: "monotonically increasing" should read "monotone"
- **Page 81, line 23**: "$\mathbb{D}$" should appear "D."
- **Page 90, line 14**: "$\lim_{x \to 0, x > 0} \frac{f(x) - f(0)}{x - 0} = -1$" should read "$\lim_{x \to 0, x < 0} \frac{f(x) - f(0)}{x - 0} = -1$."

- **Page 90, line 1 and Page 91, line 1**: "$\ldots + x_0^{n-2} + x_0^{n-1}$" should read "$\ldots + xx_0^{n-2} + x_0^{n-1}$."
- **Page 94, #3**: The function value $f(0)$ is defined twice.
- **Page 99, (4.8)**: "$x - x$" should read "$x - x_0$" in two denominators.
- **Page 107, line 8**: "$x_0 < x_0 + \delta$" should appear "$x_0 < x < x_0 + \delta$"
- **Page 107, line 15**: for "In section 9.5," write "In Section 9.6,"
- **Page 112, line 3**: "$g^{(n)}(x_0) = n!$" should appear "$g^{(n)}(x) = n!$"
- **Page 112, line 14**: "$\frac{f^{(n)}(x_n)}{g^{(n)}(x_0)}$" should appear "$\frac{f^{(n)}(x_n)}{g^{(n)}(x_n)}$"
- **Page 120, line 15**: for "inverse function $\mathbb{R}$." write "inverse function on $\mathbb{R}$."
- **Page 128, line 11**: "$C(2z) \leq 0$" should appear "$C(2z) < 0$."
- **Page 142, line 1**: for "... 1988), a clear ..." write "... 1988), is a clear ..."
- **Page 143**: Under 6.14 you should refer to Darboux sums, not Riemann.
- **Page 144, line 10**: the second "(6.19)" should be "(6.20)."
- **Page 145, line 3**: for "$[a, b] : \mathbb{R} \to \mathbb{R}$" write "$f : [a, b] \to \mathbb{R}$."
- **Page 149, #4b.**: for "$(b - a)/2$" write "$(b^2 - a^2)/2$"
- **Page 150, line 10**: for "The f is" write "Then f is."
- **Page 152, line 5**: for "$L(f, P_n)$" write "$U(g, P_n)$."
- **Page 152, line 12**: for "$\ldots \leq L(f, P) + U(g, P)$." write "$\ldots \leq U(f, P) + U(g, P)$."
- **Page 153, line 1**: for "$\ldots \leq U(f + g, P_n) \leq L(f, P_n) + U(g, P_n)$." write "$\ldots \leq U(f + g, P_n) \leq U(f, P_n) + U(g, P_n)$."
- **Page 153, line 21**: $U(\alpha f, P_n)$ Formula is in conflict with formulas 6.31. To avoid that, add the following statement: "The above formula is consistent with formula 6.31 because $U(\alpha f, P) = L(\alpha f, P)$ for all P if $\alpha = 0$."
- **Page 156, lines 10–12**: for "$[x_{i-1} - x_i]$" write "$[x_i - x_{i-1}]$"
- **Page 160, line 2**: for "Section 7.4." write "Section 7.3."
- **Page 162, line 8**: for "$L(f, P)$" write "$L(f', P)$."
- **Page 162, line 8**: for "$R(f, P)$" write "$U(f', P)$."
- **Page 164, #3**: for "$\int_a^b f = 4$" write "$\int_2^6 f = 4$"
- **Page 169, line 7**: "from bottom(7.2)" should be "(7.1)"
- **Page 169, line 8**: "from bottom(7.3)" should be "(7.2)"

- **Page 180**: Possible typo: I would delete $H(d) = 0$. Not needed in argument, and not proved. It is really necessary to change 4.19 slightly.
- **Page 187, line 11**: for "index $i \geq 1$" write "index i such that $1 \leq i \leq n$"
- **Page 189, #8**: for "Supose" write "Suppose"
- **Page 191**: In the caption of Figure 7.2; Reads "... trapezoid ..." rather than "... trapezoid ..."
- **Page 201, line 4**: for "$x = 0$" write "$x_0 = 0$"
- **Page 201, line 8**: for "$x = 0$" write "$x_0 = 0$"
- **Page 201, line 12**: for "$x = 0$" write "$x_0 = 0$"
- **Page 201, line 7**: for "$x = 1$" write "$= 1$"
- **Page 202, line 10**: for "strictly increasing ..." write "strictly decreasing ..."
- **Page 202, line 1**: for "at $x = 0$" write "at $x_0 = 0$."
- **Page 203, line 10**: for "$(x - x_0)^n$" write "$(x - x_0)^{n+1}$."
- **Page 205, line 3**: from bottom "$n > 4$" should appear "$n \geq 4$."
- **Page 206, line 1**: for "$\ln(n + 1) = \ln 1$" write "$\ln(n + 1) - \ln 1$."
- **Page 217, line 6**: for "number n" write "number k"
- **Page 221, line 10**: for "about $x = 0$" write "about $x_0 = 0$."
- **Page 225, line 6**: for "$1 \leq k \leq n$" write "$0 \leq k \leq n$."
- **Page 233, line 2**: for "for index" write "for every index"
- **Page 235, line 9**: for "$(0, c)$" write "$(0, b)$"
- **Page 240, line 2**: for "$\lim_{n\to\infty} \left(\frac{a_k}{b_k}\right)$" write "$\lim_{k\to\infty} \left(\frac{a_k}{b_k}\right)$"
- **Page 241, line 12**: for "... value is 1." write "... value is 1,"
- **Page 241, Fig. 9.2**: for "... $\lim_{n\to\infty} 1^n = 0$." write "... $\lim_{n\to\infty} 1^n = 1$."
- **Page 242, line 1**: for "... natural number k" write "... integer k."
- **Page 243, line 6**: from bottom "$2/N < x$."
- **Page 243, line 8**: for "... number n,..." write "number $n \geq 2, \ldots$"
- **Page 243, line 9**: for "$f_n(0) = f(2/n) = \cdots$" write "$f_n(0) = f_n(2/n) = \cdots$"
- **Page 243, line 10**: for "and $[2/n, 0]$" write "and $[2/n, 1]$."
- **Page 243, Fig. 9.4**: for "$\left(\frac{1}{n}, 1\right)$" write "$\left(\frac{1}{n}, n\right)$."

- **Page 251, line 4**: for "$4[b-a]$" write "$[4(b-a)]$"
- **Page 251, line 8**: for "$6[b-a]$" write "$[6(b-a)]$"
- **Page 257, line 4**: for "Cauchy on A" write "Cauchy on A"
- **Page 265, Fig. 9.6**: left figure : for "$(l, 2l)$" write "(l, l)."
- **Page 265, Fig. 9.6**: Two comments: (1) It would be nice to use the same script l as in the surrounding text. (2) It would be nice if the graphs had the same scales for both x- and y-axes.
- **Page 266, line 16**: for "$\sum_{k=1}^{\infty} h_k(x)$" write "$\sum_{k=0}^{\infty} h_k(x)$"
- **Page 279, line 6**: for "$\text{dist}(\mathbf{u}, \mathbf{u'})$ and" write "$\text{dist}(\mathbf{u}, \mathbf{u'}) = 0$ and"
- **Page 286, line 1**: from bottom "$\bigcap_{i=1}^{k} c_i$" should appear "$\bigcup_{i=1}^{k} c_i$"
- **Page 302, line 7**: for "$A : \mathbb{R} \to \mathbb{R}$" write "$f : A \to \mathbb{R}$."
- **Page 324, line 8**: for "$f : \mathbb{R} \to \mathbb{R}$" write "$f : I \to \mathbb{R}$."
- **Page 355, line 4**: Is "$\mathbf{e}_i$" defined in the text (other than p. 281, H.W. #2)?
- **Page 373, line 11**: for "$\left(\frac{1}{k}!\right)$" write "$\left(\frac{1}{k!}\right)$."
- **Page 375, line 8**: for "h" write "$\mathbf{h}$"
- **Page 391, line 8**: for "$\nabla f(x) = 0$" write "$\nabla f(x) = \mathbf{0}$"
- **Page 452, line 3**: "Since the point $(\mathbf{x_0}, \mathbf{y_0})$ belongs to V" should be replaced by "Since the point $(\mathbf{x_0}, \mathbf{0})$ belongs to V."
- **Page 474, line 10**: the word "integrable" comes before it is defined (p. 475).
- **Page 479, lines 14–15**: for "in any one of the $\mathbf{P}_k(\mathbf{J})$'s" write, perhaps, "in all of the $\mathbf{P}_k(\mathbf{J})$'s"
- **Page 479, line 16**: for "$\sum_{\mathbf{J} \text{ in } \mathbf{p}} U(\cdots) - L(\cdots)$" write "$\sum_{\mathbf{J} \text{ in } \mathbf{p}} [U(\cdots) - L(\cdots)]$."
- **Page 479, lines 14–21**: It does not seem that $\mathbf{P}_k$ can be chosen as indicated. One suggestion is to: Let $\mathbf{P}_k^*$ be the partition of $\mathbf{I}$ induced by the $\mathbf{P}_k(\mathbf{J})$'s (By this we mean that for all the $\mathbf{J}$'s in a common "row" of $\mathbf{P}$, we form the union of all the partition points of a common edge of the corresponding $\mathbf{P}_k(\mathbf{J})$'s. This union then forms one part of the partition $\mathbf{P}_k^*$ for that corresponding edge.) It should be clear that for each $\mathbf{J}$, $\mathbf{P}_k^*(\mathbf{J})$ is a refinement of $\mathbf{P}_k(\mathbf{J})$ so that

$$U(f, \mathbf{P}_k^*(\mathbf{J})) - L(f, \mathbf{P}_k^*(\mathbf{J})) \leq U(f, \mathbf{P}_k^*(\mathbf{J})) - L(f, \mathbf{P}_k^*(\mathbf{J}))$$

for all $\mathbf{J}$ and hence

$$\begin{aligned} U(f,\mathbf{P}_k^*) - L(f,\mathbf{P}_k^*) &= \sum_{\mathbf{J}} \left[U\left(f,\mathbf{P}_k^*\left(\mathbf{J}\right)\right) - L\left(f,\mathbf{P}_k^*\left(\mathbf{J}\right)\right)\right] \\ &\leq \sum_{\mathbf{J}} [U(f,\mathbf{P}_k(\mathbf{J})) - L(f,\mathbf{P}_k(\mathbf{J}))] \\ &< m\frac{1}{mk} \\ &= \frac{1}{k} \end{aligned}$$

Thus,

$$\lim_{k\to\infty} [U(f,\mathbf{P}_k^*) - L(f,\mathbf{P}_k^*)] = 0,$$

And therefore, by the Archimedes-Riemann Theorem, the function f is integrable on $\mathbf{I}$.

- **Page 479, line 19**: for "$-L(f,\mathbf{P}_k] =$" write "$-L(f,\mathbf{P}_k)] =$"
- **Page 488, line 5**: for "vol $\mathbf{J}'$" write"vol $\mathbf{J}_i'$" (twice).
- **Page 488, line 11**: for "For positive number a and b, show that the ellipse" write "Show that the set"
- **Page 488, line 7**: for "that the ellipsoid" write "that the set"
- **Page 489, line 6.7**: for "in the interior of $\mathbf{J}$" write "in the interior of $\mathbf{I}$"
- **Page 491, line 2**: for "$= \int_{\mathbf{J}} \hat{f}$," write "$= \int_{\mathbf{I}_1} \hat{f}$,"
- **Page 493, line 15**: for "$\{(\mathbf{x}, g(\mathbf{x}))$" write "$\{(\mathbf{x}, g(\mathbf{x}))\ldots$"
- **Page 499, line 10**: for "(19.3)" write "(19.1)"
- **Page 500, line 2**: for "of m_i and M_i" write "of M_i"
- **Page 519, Problem 10**: replace "R^2" by "R^n"