MATH 141 – CALCULUS II
MIDTERM EXAM # 4

Instructions. Answer each question on a separate answer sheet. **Show all your work and justify your answers**. Be sure your name, section number, and problem number are on each answer sheet, and that you have copied and signed the honor pledge on the first answer sheet. You may *not* use calculators, notes, or any other form of assistance on this exam.

(1) (30 pts)

(a) Suppose $\lim_{n\to\infty}(na_n) = 2$. Does $\sum_{n=0}^{\infty} a_n$ converge? Does $\sum_{n=0}^{\infty} a_n^2$ converge?

(b) Compute the sum: $\sum_{n=0}^{\infty}(-1)^n \frac{2^n}{3^{n+1}}$

(c) Show that $\sum_{n=1}^{\infty}(-1)^n \frac{n+1}{n^2+n+1}$ converges.

(2) (30 pts) Determine the radius of convergence each of the following power series.

(a) $\sum_{n=1}^{\infty} \frac{n^{2n}}{(2n)!}x^n$

(b) $\sum_{n=1}^{\infty}(-1)^n \frac{(\ln n)^3}{n^2}x^n$

(3) (30 pts) Find the Taylor series expansions about the point $x = 0$ of the functions:

(a) $f(x) = xe^{-x^2}$

(b) $f(x) = \int_0^x \frac{dt}{2-t^2}$

(4) (10 pts) Compute $f^{(10)}(0)$ (the tenth derivative at zero), where

$$f(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{4^n(n!)^2}$$

Date: April 29, 2011.