An On-line Robust Stabilizer

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1 Introduction

The paper presents an adaptive robust discrete solution for a stabilization problem. As an application of this solution, a SAS (Stability Augmentation System) for an aircraft is presented.

The general scheme of this system is given in the figure 1. Throughout this paper two hypothesis are assumed:

1) Full-information about the state of the system ($y_k = x_k$);

2) The Identification block gives the best fitting of the linearized time-varying nonlinear system (i.e. $x_{k+1} = A_k x_k + B_k u_k$). (see for instance [1])

The sample time of the control loop is much less than that of the identification and optimization loop. This allows on-line identification and optimization procedures.

A convenient criterion to be optimized is sought such that the control (u_k) will be given by a linear state-feedback: $u_k = F_k x_k$. This criterion will include two terms: one involving the performance requirements and the other the stability robustness:

$$C_{PR} = \lambda C_P + C_R$$

where λ is a weighting parameter.

2 The Optimization Problem

The feedback matrix F_k will be given by a Riccati equation, but instead of a classical Discrete Algebraic Riccati Equation (DARE) a modified DARE will be used such that the eigenvalues of the stable closed-loop matrix are to be within some disk included in the unit disk (according to reference [2] this is called the D-pole assignment problem). Let us consider the disk $D(\alpha, r)$ of center α and radius r in the complex plane such that α is a real number and $|\alpha| + r \leq 1$ (i.e. $D(\alpha, r) \subset D(0,1)$). In order to state the criterion, the following DARE is considered:

$$\tilde{A}_k^T K_k \tilde{A}_k - K_k - \tilde{A}_k^T K_k B_k (R + B_k^T K_k B_k)^{-1} B_k^T K_k \tilde{A}_k + Q = 0 \tag{1}$$

where: $\tilde{A}_k = (A_k - \alpha I)/r$ is the modified A-matrix and $Q, R > 0$ are positive matrices related to the quadratic cost:

$$I = \sum_{j \geq k} (x_j^T Q_k x_j + u_j^T R u_j) \tag{2}$$

with:

$$\begin{aligned} Q_k = Q + \tilde{A}_k^T K_k \tilde{A}_k - A_k^T K_k A_k + A_k^T K_k B_k (R + B_k^T K_k B_k)^{-1} B_k^T K_k A_k \\ - \tilde{A}_k^T K_k B_k (R + B_k^T K_k B_k)^{-1} B_k^T K_k \tilde{A}_k \end{aligned} \tag{3}$$

and the linear dynamics: $x_{j+1} = A_k x_j + B_k u_j \; j \geq k$.

It is known that there exists a unique stabilizable solution $K_k = K_k^T > 0$ of (1) (see [3]). Let us set:

$$F_k = -r(R + B_k^T K_k B_k)^{-1} B_k^T K_k \tilde{A}_k \tag{4}$$

and:

$$A_{s,k} = A_k + B_k F_k \tag{5}$$

Since $\Lambda(\tilde{A}_k + B_k F_k / r) \subset D(0,1)$ one can obtain:

$$\Lambda(A_{s,k}) \subset D(\alpha, r) \tag{6}$$

(here $\Lambda(\cdot)$ denotes the eigenvalues set of the matrix $\cdot$).

Now the criterion to be minimized could be stated as follows:

· the performance part:

$$C_P = x_k^T K_k x_k \tag{7}$$

· and the stability robustness part:

$$C_R = x_k^T A_{s,k}^T A_{s,k} x_k = \| A_{s,k} x_k \|^2 \tag{8}$$

The first part of the criterion is related to the quadratic cost (2) of the dynamics and the matrices Q and R are chosen to fulfil the performance requirements.

The second part gives an ∞-norm bound along the trajectory: the goal is not to minimize the H_∞-norm of A_s (which is $\max_{\|x\|=1} \| A_s x \|$) but just the $\| A_s x \|$ along the trajectory; since it is not known a priori the trajectory, in an on-line solution only $\| A_s x \|$ with x taken to be the actual state ($x = x_k$) is to be minimized. The form of C_R is suggested by the constrained stability measure introduced in reference [4] taking for G the real trajectory of our system and also $P = I$ in the formula (17) of the cited paper.

Putting these two expressions together in the criterion one can obtain a trade-off between performance and stability. Indeed, while to minimize the first part means to approach the unit circle (this is the solution of the LQ problem), the second part leads to minimizing the radius of the circle around same center (depending on x_k). The optimum of the criterion (which is the minimum) gives us the solution.

The criterion is:

$$C_{PR} = x_k^T (\lambda K_k + A_{s,k}^T A_{s,k}) x_k \tag{9}$$

where K_k is given by (1) and $A_{s,k}$ by (5) (and (4)). The Optimization Problem is then:

$$\min_{\substack{\alpha, r \\ r \geq 0\ ,\ |\alpha| + r \leq 1}} \quad C_{PR}(x_k, A_k, B_k; \alpha, r) \tag{10}$$

3 The Algorithm

Assuming that the model (A_k, B_k) is slowly time-varying, an on-line algorithm to solve (10) is proposed. The idea is not to solve (1) directly but to approach recursively to the solution using the fix point method.

To initialize the algorithm it is supposed that an initial estimation (A_0, B_0) of the model and K_0 which is the exact solution of DARE (1) and minimizes (10) are known. Then, at the kth step:

Given: x_k, A_k, B_k, K_{k-1} (and Q, R)

Set: $U = (R + B_k^T K_{k-1} B_k)^{-1} B_k^T K_{k-1}$

Search the minimum :

$$\min_{\substack{\alpha, r \\ r \geq 0\ ,\ |\alpha| + r \leq 1}} \qquad (\tilde{C} = x_k^T(\lambda K_k + A_{s,k}^T A_{s,k}) x_k)$$

subject to the relations:

$$\begin{aligned} \tilde{A}_k &= (A_k - \alpha I)/r \\ F_k &= -r\, U \tilde{A}_k \\ A_{s,k} &= A_k + B_k F_k \\ K_k &= \tilde{A}_k K_{k-1} (A_{s,k} - \alpha I)/r + Q \end{aligned}$$

using a gradient procedure with convex restrictions (given by α and r) and variable step.

The control: $u_k = \hat{F}_k x_k$ □

4 Simulation Results

The longitudinal dynamics of the airplane is given in [5] and has the form:

$$\begin{aligned} \frac{dv}{dt} &= -p\frac{S}{m}(C_D - C_T \cos\alpha) - g\sin\gamma \\ \frac{d\gamma}{dt} &= p\frac{S}{mv}(C_L + C_T \sin\alpha) - g\frac{\cos\gamma}{v} \\ \frac{dq}{dt} &= p\frac{S\,C}{I_y} C_M \\ \frac{d\theta}{dt} &= q \\ \frac{dH}{dt} &= v\sin\gamma \end{aligned} \tag{11}$$

where v is the airspeed, p is the dynamic pressure, S is the wing area, C_D is the drag coefficient, C_L is the lift coefficient, C_M is the moment coefficient, C is the chord, I_y is the moment of inertia, γ is the flight path angle, q is the pitch rate, α is the angle of attack, θ is the attitude angle, H is the altitude.

The problem is to bring the system to follow the programmed trajectory given by the attitude angle $\theta_p(t)$. Then the model state (x_k) will contain the tracking error and its derivatives with respect to t. This nonlinear model can be linearized arround the actual error and can be brought into a controllable and observable parametrized state space model of the form:

$$x(k+1) = A(\xi_k)x(k) + B(\xi_k)u(k) \quad , \quad x(0) = x_0$$

where the control u is given by the elevator command and linearly modifies the moment, lift and drag coefficients (C_M, C_L, C_D). The actual values of parameters A and B are obtained from the Identification block. The open-loop model is unstable. The dimension of the model has been chosen $n = 3$. The state of the system is then $x = (e = \theta - \theta_p, \dot{e}, \ddot{e})$. In the criterion (2) we have taken $Q = I_3$, $R = 2$ and the weighting parameter $\lambda = 0.02$.

For a reference trajectory θ_p pictured with dash line an attitude angle, both drawn in figure 2, is obtained. In figure 3 is represented the control variable which is the elevator deflection. In the first second the Identification has caused a lot of oscillations for the computed control. Moreover, it has been simulated gust as in [6]. One could see that the system tracks well the reference trajectory (this achieves the performance requirement). The presence of gust does not generate instability (this suggests the achievement of the robustness properties). The value of λ has been chosen as above in order to obtain the same order for the two terms C_P and C_R in the criterion (9).

As a conclusion, our controller tracks the desired trajectory and achieves a trade-off between the performance requirements and the robustness of the stability.

5 Conclusions

In this work an on-line solution for a robust stabilization problem is presented. As an application, a Stability Augmentation System for an aircraft is realized and the results are discussed.

The idea behind the robustness criterion is to place the poles of the closed-loop system within some disk included in the unit disk such that the criterion is minimized. The freedom degrees are in this case the radius and the position of the center of the disk. Since α must be a real number, one could see that only symmetric disks with respect to the real axis are allowed.

The criterion to be minimized has two parts (see (9)): one involving some performance requirements, related to the stabilizable solution of a certain modified DARE (equation (1)) and another part giving the robustness of the stabilized system. We stress out that the stability robustness part (C_R from (8)) is given by the norm of a state vector and not by the norm of a transfer matrix. This part of the criterion has been chosen like this because of the adaptivity of the solution.

The algorithm presented here to solve the optimization problem uses the fix point method in order to solve the modified DARE. This fact allows an on-line implementation with a faster adaptivity.

In the last section the simulation results of the SAS are presented and discussed.

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