Modeling and Results for a Mach-Zehnder Chaotic System

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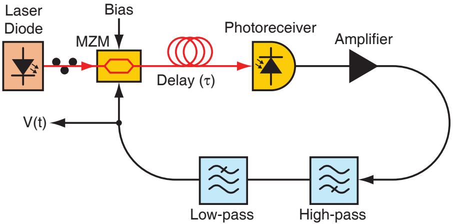
Why bother?

- Reduce 'Experimentation' time
- Comfirm Theories
- Direct Research
- Isolate errors
- Find new results

System Overview

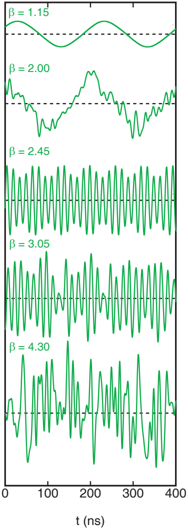


$$x(t) + \tau \dot{x}(t) + \frac{1}{\theta} \int_{t_o}^{t} x(s) ds = \beta cos^2[x(t-T) + \phi]$$

- $x(t)$ is normalized RF voltage
- τ is the low pass filter time constant
- θ is the high pass filter time constant
- T is the time delay in the loop
- β is the feedback strength
- ϕ is the phase offset in nonlinearity

[Kouomou, Thesis]

Modeling the system



[Cohen et. al., PRL]

- Traditional numeric methods can be applied
- Generates a variety of dynamics
- What can improve?
  - Speed
  - Versatility of Model
- This Gets us:
  - Ability to explore wider parameter choices
  - New avenues of research
  - New results

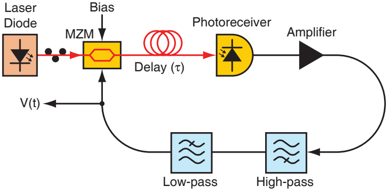
Details of Improvements

- Improving Speed
  - Change Integration $\longrightarrow$ Discrete Map
  - Incorporate matrix math vs. multi-steps
- Improving Versatility
  - Allow more filter types/orders
  - Explicit variable differation
  - Simple expansion to multiple systems

Comparing Differentials vs. State-Space

Differential Form

$$x(t) + \tau\dot{x}(t) + \frac{1}{\theta}\int_{t_o}^{t} x(s)ds = y(t)$$



Discrete State-Space Form

$$\mathbf{u}_1[n+1] = \mathbf{A}\mathbf{u}_1[n] + \mathbf{B}\cos^2(\mathbf{C}\mathbf{u}_1[n-k] + \phi)$$

Equations for Coupled Mach-Zehnder Loops

(one Mach-Zehnder loop)

$$\mathbf{u}_1[n+1] = \mathbf{A}\mathbf{u}_1[n] + \mathbf{B}\cos^2(\mathbf{C}\mathbf{u}_1[n-k] + \phi)$$

(coupled Mach-Zehnder loops)

$$\begin{aligned}\mathbf{u}_1[n+1] = \mathbf{A}\mathbf{u}_1[n] &+ (1-\gamma_1) * \mathbf{B}\cos^2(\mathbf{C}\mathbf{u}_1[n-k_1] + \phi_1) \\ &+\gamma_1 \quad * \mathbf{B}\cos^2(\mathbf{C}\mathbf{u}_2[n-k_2] + \phi_2)\end{aligned}$$

$$\begin{aligned}\mathbf{u}_2[n+1] = \mathbf{A}\mathbf{u}_2[n] &+ (1-\gamma_2) * \mathbf{B}\cos^2(\mathbf{C}\mathbf{u}_2[n-k_2] + \phi_2) \\ &+\gamma_2 \quad * \mathbf{B}\cos^2(\mathbf{C}\mathbf{u}_1[n-k_1] + \phi_1)\end{aligned}$$

Milestones

- Implementation and Verification of individual simulations — November 1^{st} (complete)
- Implementation and Verification of final simulation — December 1^{st} (complete)
- Generation of new results — Feburary 1^{st} (complete)
- Further Expansion and Use of Code — Ongoing

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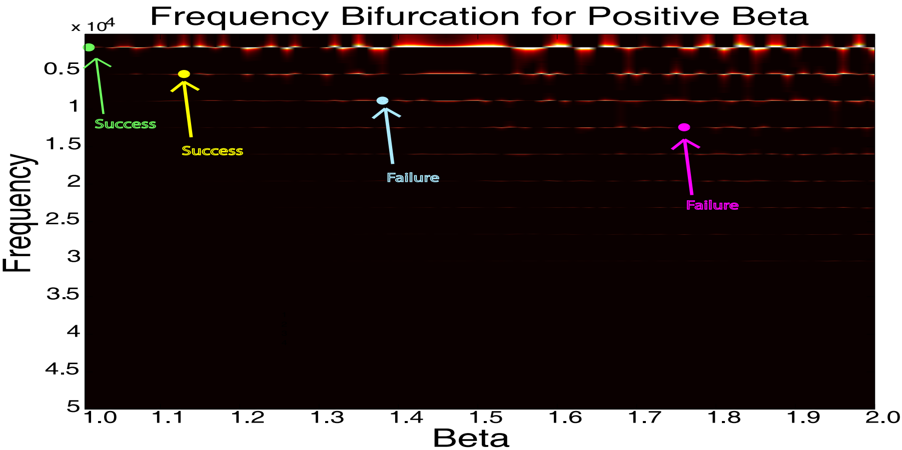
Validation Plan

Stage 1: Single Loop	
• Characteristic Curves from Kouomuo[1]	Success
• Bifurcation Points from Kouomuo	Partial Success
Stage 2: Coupled Lorenz	
• Conditions by Anishchenko [4]	Success
Stage 3: Coupled Mach-Zehnders	
• Open Loop: Argysis [5]	Success
• Symmetric 50/50 coupling: Piel [6]	Success

Comparision of Analytic to Simulated Results

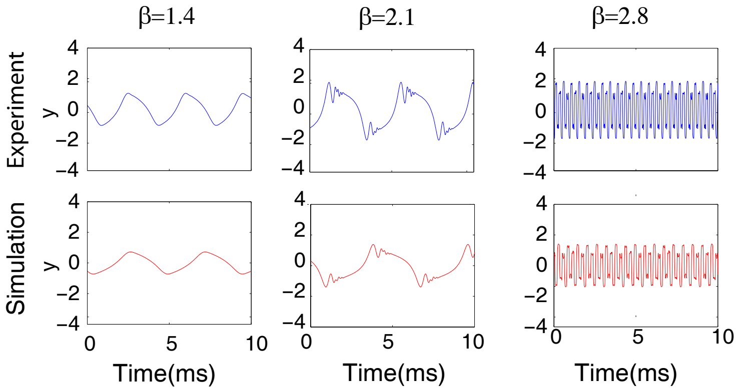


Kouomou predicts the following bifurcations:

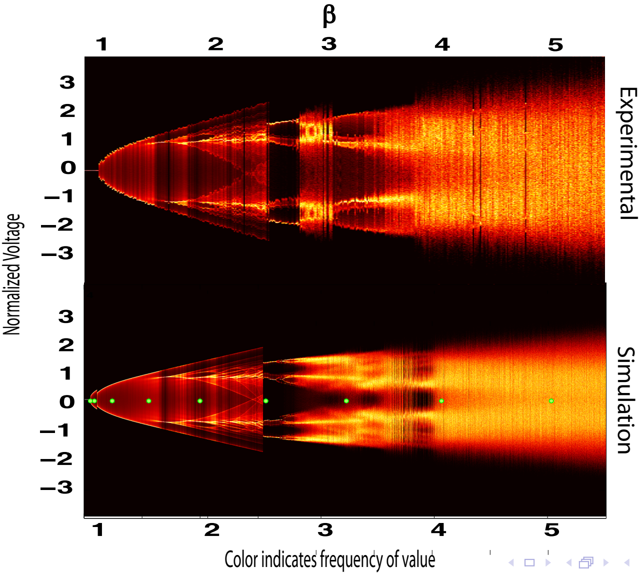
Hopf Bifurcation Points

$$\beta_k = (-1)^{k+1}[1 + \frac{(\epsilon R^2 - k^2\pi^2)^2}{2k^2\pi^2 R^2}] \qquad \omega_k = k\frac{\pi}{R}$$

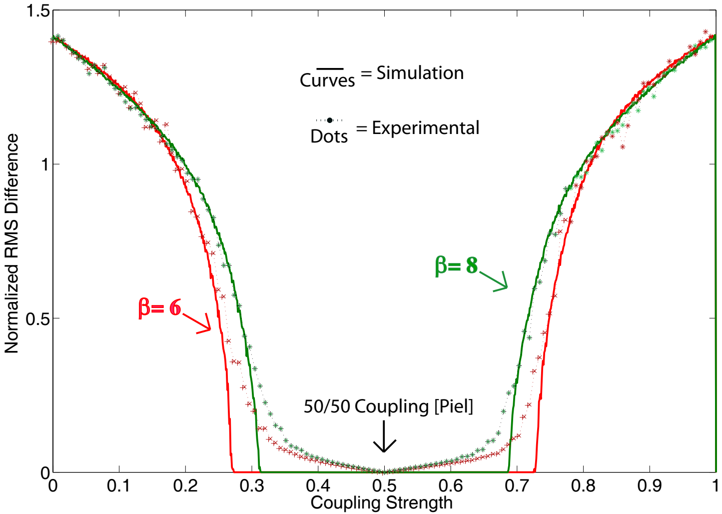
Comparision of Experimental to Simulated Time Series



Comparision of Experimental to Simulated Bifurcation



Synchronization of Coupled Systems



What We Want

Looking for:

- Affects of measurement error
- Exploration of new variables
- Analysis of larger parameter space

Equations for Coupled Mach-Zehnder Loops

(one Mach-Zehnder loop)

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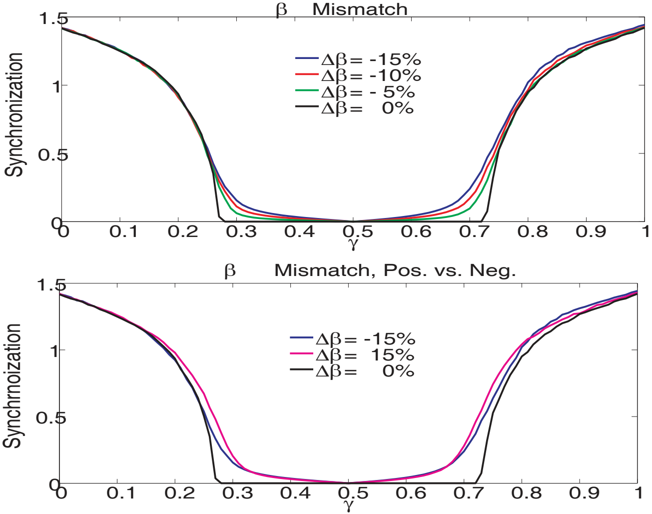
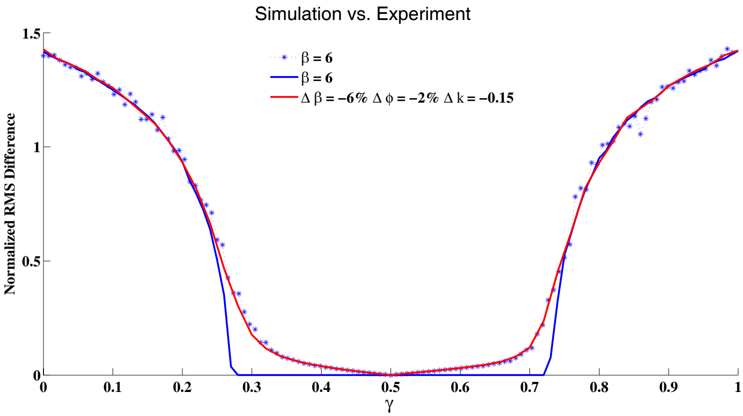
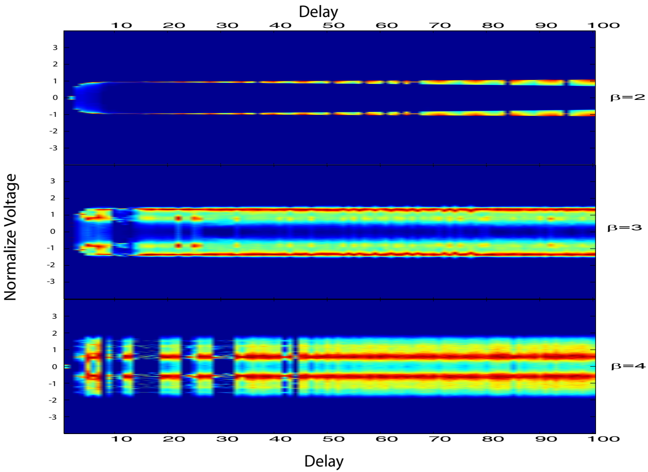
Mismatch 1


β Mismatch
Synchronization
Δβ= -15%
Δβ= -10%
Δβ= - 5%
Δβ= 0%
γ
β Mismatch, Pos. vs. Neg.
Synchrnoization
Δβ= -15%
Δβ= 15%
Δβ= 0%
γ

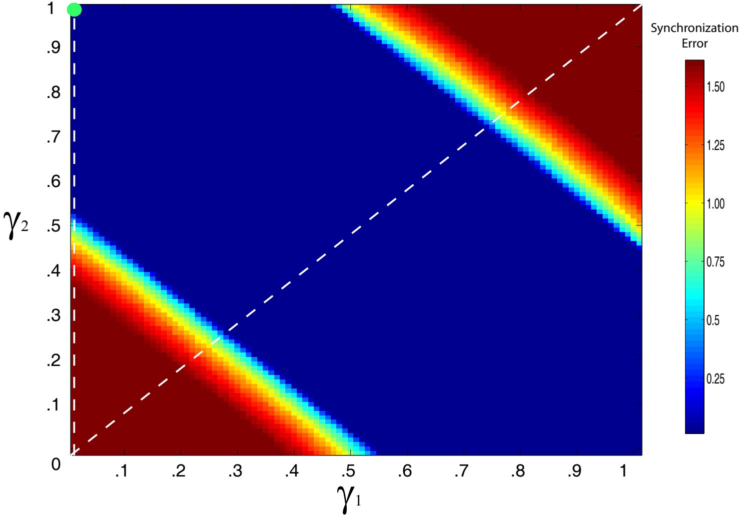
Matching the Experiment



Voltage Bifurcations in Delay (k)



γ_1 vs γ_2



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