Homework #1
Due: Thursday, February 3, 2011

1. Compute the following integrals:

$$\int_0^1 \log(x)dx$$

where *log(x)* denotes the natural logarithm of *x* also known as *ln(x)*, and

$$\int_1^\infty \frac{1}{x^a}dx$$

for a fixed parameter $a>1$. What happens for $a=1$?

2. Consider the following function:

$$f : R \to R \ , \ f(x) = \begin{cases} x\log(x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

   a. Is this function continuous on *R* (that is of class C^0)?
   b. Is this function differentiable on *R* ?
   c. Is this function of class C^1 (that is continuous, differentiable, and has continuous first derivative)?
   d. What is the largest integer *k* so that *f* is of class C^k ?

Note: A function is said of class C^k if it is *k* times differentiable, and the k^{th} derivative is continuous. Thus a function is of class C^0 if it is continuous; a function is of class C^1 if it is continuous, differentiable, and has continuous first derivative; a function is of class C^2 if it is continuous, differentiable, it first derivative is continuous, differentiable, and its second derivative is continuous. In general a function *f* is of class C^k if its derivative *f'* is of class C^{k-1}.

3. Consider the following functions:

$$g : R \to R \ , \ g(x) = \begin{cases} \sin\left(\frac{1}{x}\right) \ , \ \text{if } x \neq 0 \\ 0 \ , \ \text{if } x = 0 \end{cases}$$

$$f : R \to R \ , \ f(x) = \int_0^x g(t)dt$$

Prove that *f* is differentiable but is not of class C^1. (See above note)

4. Recall Euler's formula $e^{i\theta} = \cos(\theta) + isin(\theta)$ and the geometric series sum formula

$$1 + z + z^2 + \cdots + z^n = \frac{1-z^{n+1}}{1-z}.$$

Solve the following equations where the unknown is θ:

   a. $1 + e^{i\theta} = 0 \ , -\infty < \theta < \infty$
   b. $1 + e^{i\theta} + e^{2i\theta} = 0 \ , -\infty < \theta < \infty$

c. $1 + e^{i\theta} + e^{2i\theta} + e^{3i\theta} = 0$, $-\infty < \theta < \infty$
d. $1 + e^{i\theta} + e^{2i\theta} + \cdots + e^{ni\theta} = 0$ where n is a fixed positive integer and $-\infty < \theta < \infty$