Homework #10
Due: Thursday, April 28, 2011

Note: Use of Matlab (or any other software) is not permitted.

I. (Exercise 7.27) Find all generalized functions *f* that satisfy each of the following inhomogeneous equations.

1. (1pt) $(x^4 - 1) \cdot f(x) = \delta(x)$;

2. (1pt) $(x^4 + 1) \cdot f(x) = \delta(x)$;

Hint: Study exercises 7.23, 7.24.

II. (Exercise 7.33, 7.36) Find the Fourier transform of each of the following generalized functions. In so doing, freely use the rules from the Fourier transform calculus together with the Fourier transforms we deduced in class.

3. (1pt) $f(x) = e^{2\pi i x} \cdot \delta(x)$

4. (1pt) $f(x) = \sin^2(\pi x)$

5. (1pt) $f(x) = x^2 \cdot \delta'(x)$

6. (1pt) $f(x) = e^{4\pi i x} \cdot sign(x)$

7. (1pt) $f(x) = \dfrac{1}{x^2 - 4}$

III. (Exercise 7.27) Find all generalized functions *f* that satisfy each of the following inhomogeneous equations.

8. (1pt) $(D^4 - 1) f(x) = \delta(x)$;

9. (1pt) $(D^4 + 1) f(x) = \delta(x)$;

Here D denotes the derivative operator. Thus $D^4 f(x) = \dfrac{d^4 f}{dx^4}(x)$.

Hint: Fourier transform the equation, solve the algebraic equation thus obtained, and then Fourier invert the solution.

IV. Find all generalized functions *f* that satisfy:

10. (1pt) $(D^2 - 1) f(x) = \sin(x)$;

Hint: Use convolution with the Green's function, or Fourier transform the equation.

Total: 10 pts (1 point each)