Homework #11
Due: Thursday, May 5, 2011

Note: Use of Matlab (or any other software) is not permitted.

I. Consider the following equation on the real line:

$$u_{tt} = u_{xx} \ , \ -\infty < x < \infty$$
$$u(x,0) = \sin(x) \ , u_t(x,0) = -\cos(x)$$

1. (1pt) Find the solution u(x,t);
2. (1pt) Plot the solution as function of x at t=0 and t=π

II. Consider the wave equation on the real line $u_{tt} = 4u_{xx}$ with initial conditions $u(x,0) = e^{-x^2/2}$, $u_t(x,0) = 2xe^{-x^2/2}$.

3. (1pt) Solve this problem for u(x,t);
4. (1pt) Plot u(x,0) and u(x,1);
5. (1pt) What is the solution of the following initial value problem (on the real line):

$$u_t = 4u_{xx} \ , \qquad u(x,0) = e^{-x^2/2}$$

III. Consider the periodic wave equation on a circle of circumference 12 (u(x+12,t)=u(x,t)):

$$u_{tt} = 9u_{xx} \ , \qquad 0 \le x \le 12$$
$$u(x,0) = \Lambda(x-6) \ , \qquad u_t(x,0) = 0$$

6. (1pt) Find the solution u(x,t)
7. (1pt) Is there a time T so that u(x,t+T)=u(x,t) ? If yes, determine the smallest positive T with this property.
8. (1pt) Plot the solution u(x,t) at t=0, t=1, and t=4 on the interval 0≤x≤12.

IV. (1pt) Solve the periodic heat equation (u(x+1,t)=u(x,t))

$$u_t = 9u_{xx} \ , \ 0 \le x \le 1$$
$$u(x,0) = \sin(6\pi x)$$

V. (1pt) Consider a vibrating string of unit length with fixed ends that satisfies the following initial value problem

$$u_{tt} = 9u_{xx}$$
$$u(x,0) = \sin(6\pi x) \ , \qquad u_t(x,0) = 0$$

Solve for u(x,t).

Total: 10 pts