# Homework #5
# Due: Thursday, March 3, 2011

**Note**: Use of Matlab (or any other software) is not permitted.

**I.** (Exercise 3.9 and 3.12) Find the Fourier transform of each of these functions:

1. $f(x) = e^{-|2x+5|}$
2. $f(x) = e^{-3x^2}$
3. $f(x) = \int_{-\infty}^{\infty} e^{2\pi i s x - s^4} ds$
4. $f(x) = \dfrac{2}{x^2 - 4x + 5}$
5. $f(x) = \dfrac{2x-4}{x^2 - 4x + 5}$

**II.** (Exercise 3.28) Use your knowledge of Fourier analysis to find a function *f* that satisfies the given integral equation:

6. $\int_{0}^{\infty} f(u)\cos(2\pi u x)du = e^{-x}\ ,\ 0 < x < \infty$
7. $\int_{0}^{\infty} f(u)\sin(2\pi u x)du = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 < x < \infty \end{cases}$
8. $\int_{-\infty}^{\infty} f(u) f(x-u) du = e^{-\pi x^2}$

**III.** (Exercise 3.29) Let *g* be a piecewise smooth function on **R** which is absolutely integrable (that is $\int_{-\infty}^{\infty} | g(x) dx < \infty$), and suppose that we wish to find such a function *f* that satisfies the differential equation:

$$-f''(x) + f(x) = g(x)\ ,\ -\infty < x < \infty$$

9. Fourier transform the differential equation and thereby show that any suitably regular solution can be written in the form

$$f(x) = \frac{1}{2}\int_{-\infty}^{\infty} e^{-|x-u|} g(u) du$$

10. Find the function *f* and sketch its graph when $g(x) = \Pi(x)$.

*Total*: 10 pts (1 point each)